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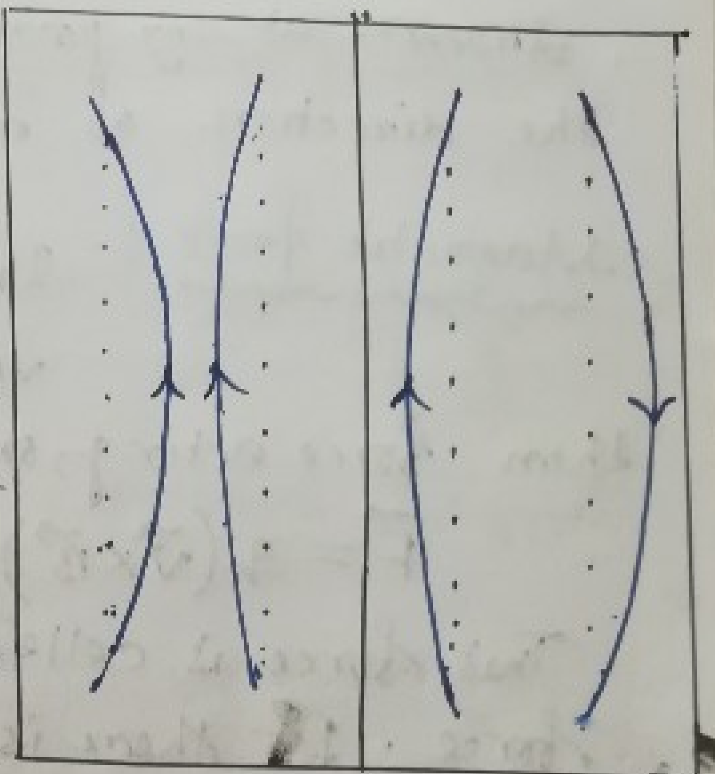
Magnetic field



It is observed that if current flow in the same direction of two parallel conducting wires then they attract each other while current flow in the opposite direction they repel. Now if we keep a test charge near the wires there is no force on it. So there is no electric field in the vicinity of two wires. But when a tiny compas is kept then it is deflected towards field direction. So the field is nothing but magnetic field.

So a stationary charge produces only electric field E in the space around it while a moving charge generates in addition a magnetic field.

It is observed if current flows in a straight conductor the magnetic field encircles around it. If we grab the wire with right hand keeping thumb in the direction of current then our fingers curl around denotes the magnetic field lines. Tangent drawn at any point along the field line denotes the direction of magnetic field at that point.

Magnetic force :- If a charge q moves with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ then force acting on it is

$$\vec{F} = q(\vec{v} \times \vec{B}) \quad \text{------------} \quad (i)$$

This force is called magnetic force or Lorentz force. If there is an electric field in addition with magnetic field then $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$

* Now if the charge moves a length dl in time dt then $dl = v\,dt$. So work done

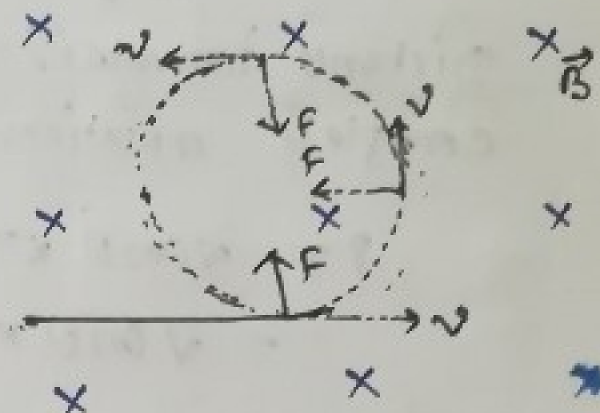
$$dW = \vec{F}\cdot\vec{dl} = q(\vec{v}\times\vec{B})\cdot\vec{dl}$$
$$= q(\vec{v}\times\vec{B})\cdot\vec{v}\,dt$$
$$= 0$$

So it is said that magnetic force do no work. Magnetic force may change the direction in which a particle moves but they can not speed up or slow it down.

* From above eqn (i) if $\vec{v}$ is parallel to $\vec{B}$ then force on the charged particle is zero.

* Then magnetic force becomes maximum when $\vec{v}$ is perpendicular to $\vec{B}$. Then force

$$F = qvB$$



Now since the force is always perpendicular to the instantaneous velocity. So the particle moves in a circular path. This force supply the centripital force to move in a circular path.

$$\therefore \quad qvB = \frac{mv^2}{r}$$

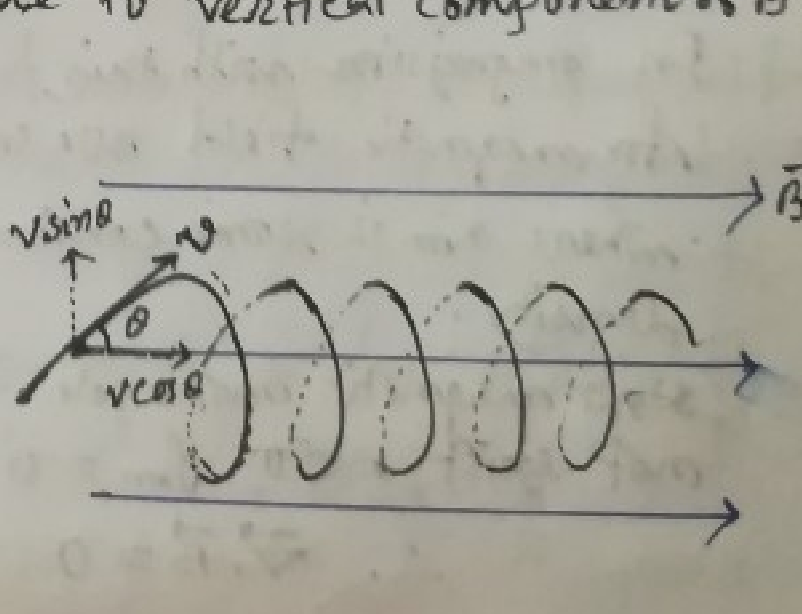
$$\text{or.} \quad r = \frac{mv}{Bq}$$

Time period of rotation $T = \frac{2\pi r}{v} = \frac{2\pi}{v} \times \frac{mv}{Bq} = \frac{2\pi m}{Bq}$

$\therefore$ frequency of rotation $n = \frac{1}{T} = \frac{Bq}{2\pi m}$

This frequency is called cyclotron frequency.

* If the charged particle enters in a magnetic field $\vec{B}$ with an angle θ then due to vertical component of $\vec{B}$ ie due to $v\sin\theta$ it moves in a circular path of radius r and due to horizontal component $v\cos\theta$ it moves along the field direction. Ultimately the path becomes helical.



So using above relation, the radius of the circular path

$$r = \frac{m(v\sin\theta)}{Bq}$$

Distance traversed along the field direction in one complete rotation is called pitch. So

$$P = v\cos\theta \times T$$

$$= v\cos\theta \times \frac{2\pi m}{Bq}$$ [Time period remains same as like the previous case as it does not depends upon the velocity]

$$= \frac{2\pi m v \cancel{\sin\theta} \cos\theta}{Bq}$$

Origin of magnetic field

~~Like a parmanent~~ Like a parmanent magnet a steady current also produce a magnetic field around it. So magnetic effect of steady currents form the subject matter of magnetostatics.

From the Gauss's law of electrostatics we know

$$\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$$, ρ being the charge density and ϵ_0 is free space permitivity.

In comparision with this,
for magnetic field we can write $\vec{\nabla}\cdot\vec{B} = K_m \rho_m$
where K_m is some constant and ρ_m is magnetic charge density.

Now magnetic monopole ie isolated magnetic pole does not exist. So $\rho_m = 0$

$$\therefore \vec{\nabla}\cdot\vec{B} = 0$$

Again, it can be shown that

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

where μ_0 is the free space permiability and J is current density.

Now taking divergence in both sides of the above eqn

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = \mu_0(\vec{\nabla}\cdot\vec{J})$$

$$\text{so, } \vec{\nabla}\cdot\vec{J} = 0$$

from continuity eqn we know that $\vec{\nabla}\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$

$$\therefore \frac{\partial\rho}{\partial t} = 0$$

ie charge density is independent of time which implies steady current.

Since $\vec{\nabla}\cdot\vec{B} = 0$

$$\therefore \oint_S \vec{B}\cdot d\vec{s} = 0 \quad \text{[using divergence theorem]}$$

The quantity $\oint_S \vec{B}\cdot d\vec{s}$ is called magnetic flux (Φ). The above equation says that magnetic flux through any closed surface is zero ie the magnetic field lines are continious.

$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow$ Gauss's law of magnetostatics in differential form

$\oint_S \vec{B}\cdot d\vec{s} = 0 \Rightarrow$ " " " " in integral "

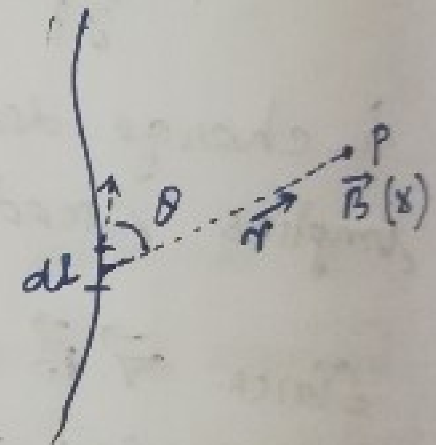
Unit: SI Unit of magnetic field vector is wb/m^2 or tesla (T). and in CGS it is Mx/cm^2 or gauss (G).

$$1\,T = 1\,wb/m^2 = \frac{10^8\,Mx}{10^4\,cm^2} = 10^4\,G.$$

SI unit of magnetic flux (Φ) is wb and Mx in CGS.

The Biot-Savart law :- Magnetic field ~~at~~ due to a current element $\vec{dl}$ at any point at a distance r when a steady current I flows through the conductor is given by

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{\vec{dl} \times \hat{r}}{r^2}$$

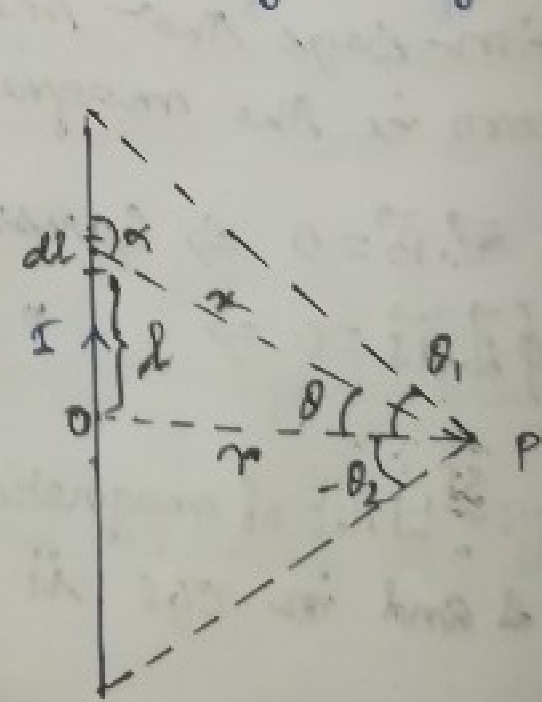


field at that point due to the whole wire is

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{dl} \times \hat{r}}{r^2}$$

* Magnetic field due to a current carring straight wire :-

Let us ~~consi~~ consider a ~~conductor~~ straight conductor carring a current I. We have to find the magnetic field at P, at a distance r from point O.



Let us consider a small element dl, at a distance l from O. Now the magnetic field at P due to this small element is

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\alpha}{x^2} \quad (x)$$

Now, $\sin\alpha = \sin(90° + \theta) = \cos\theta$

$\frac{l}{r} = \tan\theta$ and $\frac{r}{x} = \cos\theta$

$dl = r\sec^2\theta\, d\theta$. $x = r\sec\theta$.

$$\therefore dB = \frac{\mu_0 I}{4\pi} \frac{r \cancel{\sec^2\theta} \cos\theta\, d\theta}{r^2 \cancel{\sec^2\theta}}$$

$\therefore$ Magnetic field at P due to the whole wire is

$$B = \frac{\mu_0 I}{4\pi r} \int_{-\theta_2}^{\theta_1} \cos\theta\, d\theta$$

$$\vec{B} = \frac{\mu_0 I}{4\pi r} (\sin\theta_1 + \sin\theta_2) \quad (x)$$

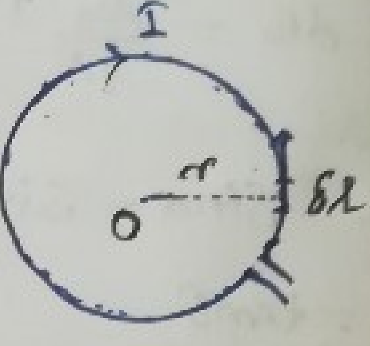
* for infinitely long current carring wire $\theta_1 = \theta_2 = \pi/2$

$$\therefore B = \frac{\mu_0 I}{2\pi r}$$

* For half infinitely long current carring wire $\theta_1 = 0°$; $\theta_2 = \pi/2$ or vice versa then

$$B = \frac{\mu_0 I}{4\pi r}.$$

* Magnetic field at the centre of a circular current carring conductor :



Let us consider a circular conductor having radius r, carries a current I as shown. Now due to the small element δl the magnetic field at the centre O is

$$\delta\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,\delta l \sin 90^\circ}{r^2} \quad (\times)$$

So, considering the whole conductor

$$B = \sum \frac{\mu_0}{4\pi} \frac{I\,\delta l}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I}{r^2} \sum \delta l$$

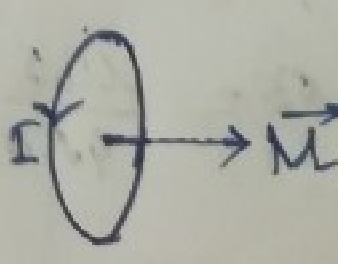
$$= \frac{\mu_0 I}{4\pi r^2} \times 2\pi r$$

$$\vec{B} = \frac{\mu_0 I}{2r} \quad (\times)$$

If there is N number of turns then the expression for magnetic field is

$$\vec{B} = \frac{\mu_0 N I}{2r} \quad (\times)$$

\# Current carring loop :-



A current carring loop is equivalent to a magnetic dipole whose dipole moment $\vec{M} = I\vec{A}$ where A is area of the loop.

If we observe the loop (as shown in the fig. above) from the left side of the loop, the direction of the current is in the clockwise direction. So left side of the loop behaves as South pole. Again if we observe it from right side then the direction of current is in the anti-clockwise direction i.e. it behaves like a north pole. So it is said that a current carring loop is equivalent to a magnetic dipole.

We can compare it with a magnetic dipole of pole strength m and length d. Then

$$IA = md$$