Economics Honours

Semester 2

Paper C3 (Microeconomic Theory II)

Cournot's Duopoly Model:

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The earliest duopoly model was developed in 1838 by the French economist Augustin Cournot. Cournot assumed that there are two firms each owning a mineral well, and operating with zero costs. They sell their output in a market with a straight-line demand curve. Each firm acts on the assumption that its competitor will not change its output, and decides its own output so as to maximise profit.

Assume that firm A is the first to start producing and selling mineral water. It will produce quantity A, at price P where profits are at a maximum, because at this point MC = MR = 0.

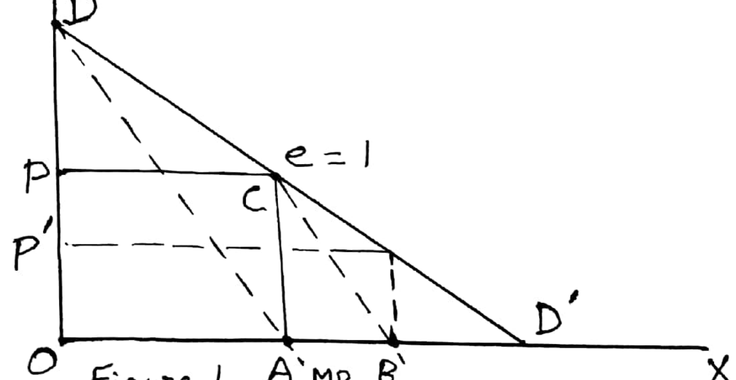

Figure 1

The elasticity of market demand at this level of output is equal to unity and the total revenue of the firm is a maximum. With zero costs, maximum R implies maximum profits, π. Now firm B assumes that A will keep its output fixed (at OA), and hence considers that its own demand curve is CD′. Clearly firm B will produce half the quantity AD′, because at this level (AB) of output (and at price P′) its revenue and profit is at a maximum. B produces half of the market which has not been supplied by A, that is, B's output is $\frac{1}{4}\left(=\frac{1}{2}\cdot\frac{1}{2}\right)$ of the total market.

Firm A, faced with this situation, assumes that B will retain his quantity constant in the next period. So he will produce one-half of the market which is not supplied by B. Since B covers one-quarter of the market, A will, in the next period, produce $\frac{1}{2}\left(1-\frac{1}{4}\right)=\frac{1}{2}\cdot\frac{3}{4}=\frac{3}{8}$ of the total market.

Firm B reacts on the Cournot assumption, and will produce one-half of the unsupplied section of the market, i.e. $\frac{1}{2}\left(1-\frac{3}{8}\right)=\frac{5}{16}$.

In the third period firm A will

continue to assume that B will not change its quantity, and thus will produce one-half of the remainder of the market, i.e. $\frac{1}{2}\left(1-\frac{5}{16}\right)$.

This action-reaction pattern continues, since firms have the naive behaviour of never learning from past ~~patterns~~ patterns of reaction of their rivals. However, eventually an equilibrium will be reached in which each firm produces one-third of the total market. Together they cover two-thirds of the total market. Each firm maximises its profit in each period, but the industry profits are not maximised.

The equilibrium of the Cournot firm may be obtained as follows:

1. The product of firm A in successive perio

period 1: $\frac{1}{2}$

period 2: $\frac{1}{2}\left(1-\frac{1}{4}\right)=\frac{3}{8}=\frac{1}{2}-\frac{1}{8}$

period 3: $\frac{1}{2}\left(1-\frac{5}{16}\right)=\frac{11}{32}=\frac{1}{2}-\frac{1}{8}-\frac{1}{32}$

period 4: $\frac{1}{2}\left(1-\frac{42}{128}\right)=\frac{43}{128}=\frac{1}{2}-\frac{1}{8}-\frac{1}{32}-\frac{1}{128}$

$$\begin{bmatrix}\text{Product of A}\\ \text{in equilibrium}\end{bmatrix}=\frac{1}{2}-\frac{1}{8}-\frac{1}{32}-\frac{1}{128}\cdots\cdots$$

$$=\frac{1}{2}-\left[\frac{1}{8}+\frac{1}{8}\cdot\frac{1}{4}+\frac{1}{8}\cdot\left(\frac{1}{4}\right)^2+\frac{1}{8}\left(\frac{1}{4}\right)^3+\cdots\right.$$

$$\begin{bmatrix}\text{Product of A}\\ \text{in equilibrium}\end{bmatrix} = \frac{1}{2} - \frac{\frac{1}{8}}{1-\frac{1}{4}} \quad \left[\begin{array}{l}\text{Sum} = \frac{a}{1-r}\\ \text{where } a = \frac{1}{8}\\ \& \ r = \frac{1}{4}\end{array}\right.$$

$$= \frac{1}{2} - \frac{\frac{1}{8}}{\frac{3}{4}}$$

$$= \frac{1}{2} - \frac{4}{24} = \frac{1}{3}$$

2. The product of firm B in successive periods

period 2: $\frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$

period 3: $\frac{1}{2}\left(1-\frac{3}{8}\right) = \frac{5}{16} = \frac{1}{4} + \frac{1}{16}$

period 4: $\frac{1}{2}\left(1-\frac{11}{32}\right) = \frac{21}{64} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$

period 5: $\frac{1}{2}\left(1-\frac{43}{128}\right) = \frac{85}{256} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256}$

$$\begin{bmatrix}\text{Product of B}\\ \text{in equilibrium}\end{bmatrix} = \frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4} + \frac{1}{4}\left(\frac{1}{4}\right)^2 + \frac{1}{4}\left(\frac{1}{4}\right)^3 + \cdots$$

$$= \frac{\frac{1}{4}}{1-\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Thus the Cournot solution is stable. Each firm supplies ⅓ of the market, at a common price which is lower than the monopoly price, but above the pure competitive price. It can be shown that if there are three firms in the industry, each will produce one-quarter of the market and all of them together will supply $\frac{3}{4}\left(=\frac{1}{4}\cdot 3\right)$ of the entire market OD'. And, in general, if there are n firms in the industry each will provide $1/(n+1)$ of the market.