Physics Honors. Semester – II; 2020.

Alokkumar De.

Theory of Errors: Systematic and Random Errors. Propagation of Errors. Normal Law of Errors. Standard and Probable Error.

Theory of Errors. **PART - II**

Some specific cases:

01. Sum

Let

x=u+a
where
x → variable dependent on the measured variable u.
a → constant.

$$\left(\frac{\partial x}{\partial u}\right)=1$$

And thus

$$\sigma_x=\sigma_u.$$

This yields, relative uncertainty as

$$\frac{\sigma_x}{x}=\frac{\sigma_u}{x}=\frac{\sigma_u}{u+a}. \qquad \text{.... ....} \quad (14)$$

02. Difference

If
x=u-a

$$\frac{\sigma_x}{x}=\frac{\sigma_u}{u-a}. \qquad \text{.... ....} \quad (15)$$

03. Weighted Sum

Let

x=au+bv
where
x → Weighted sum of u and v..
a, b → constants.

$$\left(\frac{\partial x}{\partial u}\right) = a \quad ; \quad \left(\frac{\partial x}{\partial v}\right) = b$$

And thus

$$\sigma_x^2 = a^2\sigma_u^2 + b^2\sigma_v^2 + 2ab\sigma_{uv}^2 \qquad \ldots \quad \ldots.. \quad \ldots.. \quad (16)$$

04. Multiplication

Let

x=auv
where
u, v → two variables.
a → constant.

$$\left(\frac{\partial x}{\partial u}\right) = av \quad ; \quad \left(\frac{\partial x}{\partial v}\right) = au$$

Thus , we can write the variance of x as

$$\sigma_x^2 = (av\sigma_u)^2 + (au\sigma_v)^2 + 2a^2uv\sigma_{uv}^2 \qquad \ldots \quad \ldots.. \quad \ldots \quad \ldots.. \quad (17)$$

which gives

$$\frac{\sigma_x^2}{x^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2} + 2\frac{\sigma_{uv}^2}{uv} \qquad \ldots. \quad \ldots.. \quad \ldots.. \quad \ldots.. \quad 18$$

05. Division

If

$$x = \frac{au}{v},$$

$$\frac{\sigma_x^2}{x^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2} - \frac{\sigma_{uv}^2}{uv} \qquad \text{.... ....} \qquad 19$$

06. Power

If

$$x = au^b,$$

$$\left(\frac{\partial x}{\partial u}\right) = abu^{b-1} = \frac{bx}{u} \qquad \text{.... ....} \qquad 20.$$

Therefore

$$\frac{\sigma_x}{x} = b\frac{\sigma_u}{u} \qquad \text{.... ....} \qquad 21$$

Normal Distribution:

The distribution of a random variable which follows Gaussian one is what is said to be normal distribution [05].

In probability theory, Normal or Gaussian or Gauss or Laplace-Gauss distribution is a type of continuous probability distribution for a real valued random variable. The general form of probability density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{.... ....} \quad (22)$$

where

μ → mean or expectation of the distribution

σ → standard deviation,

σ^2 → variance of the distribution.

Sometimes a normal distribution (Fig. 03) is informally called a *bell curve*.

In the special case when μ = 0 and σ = 1, one obtains the simplest case of normal distribution. This is known as the standard normal distribution. Probability density function for the same is described by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \quad \text{.... ....} \quad (23)$$

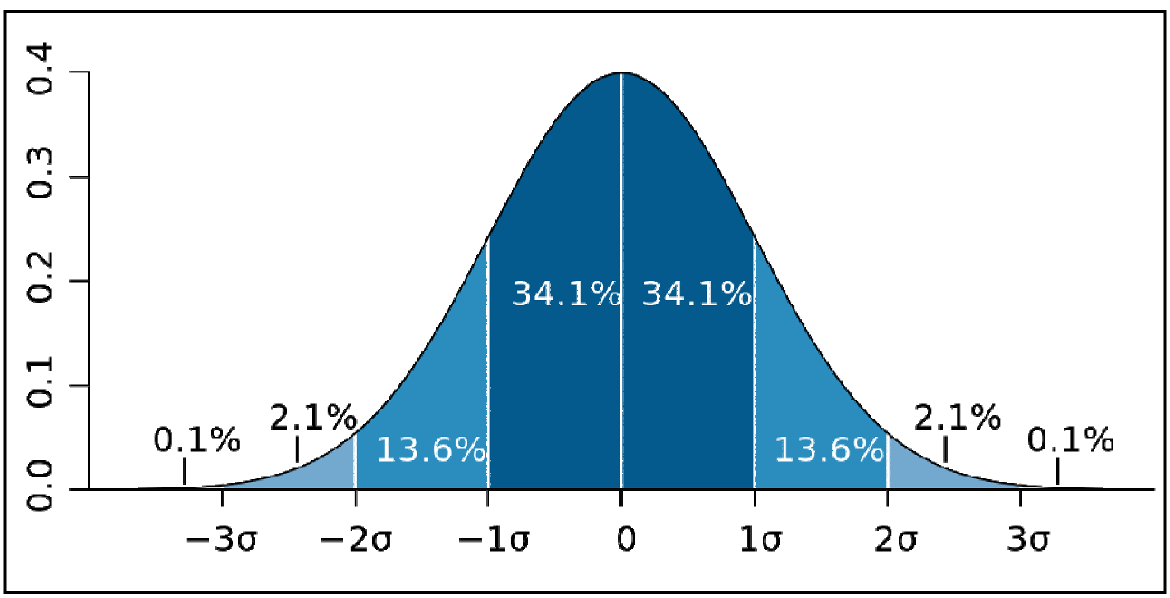

Fig. 03." For the normal distribution, the values less than one standard deviation away from the mean account for 68.27% of the set; while two standard deviations from the mean account for 95.45%; and three standard deviations account for 99.73%."[05]

Standard error:

"a measure of the statistical accuracy of an estimate, equal to the standard deviation of the theoretical distribution of a large population of such estimates."

- Oxford

The **standard error** (**SE**) of a statistic (usually an estimate of a parameter) is the standard deviation of its sampling distribution or an estimate of that standard deviation. If the parameter or the statistic is the mean, it is called the **standard error of the mean** (**SEM**).[06]

" the relationship between the standard error and the standard deviation is such that, for a given sample size, the standard error equals the standard deviation divided by the square root of the sample size. In other words, the standard error of the mean is a measure of the dispersion of sample means around the population mean."[06]

The standard error of the mean (SEM) can be expressed as

$$\sigma_x = \frac{\sigma}{\sqrt{n}} \qquad \text{.... ....} \qquad (24)$$

where

σ → standard deviation of the population and

n →number of observations (size) of the sample.

The standard error of the mean is usually estimated as

$$\sigma_x \approx \frac{s}{\sqrt{n}} \qquad \text{.... ....} \qquad (25)$$

Since the population standard deviation is is seldom known. In the above equation (25)

s is the sample standard deviation, i.e., the sample-based estimate of the standard deviation of the population.

Probable error:

"In statistics, **probable error** defines the half-range of an interval about a central point for the distribution, such that half of the values from the distribution will lie within the interval and half outside." [07, 08]

References:

[01] https://www.physics.umd.edu/courses/Phys276/Hill/Information/Notes/ErrorAnalysis.html

[02] https://sciencing.com/difference-between-systematic-random-errors-8254711.html

[03] https://en.wikipedia.org/wiki/Propagation_of_uncertainty ; and the references therein

[04] circuit globe.com

[05] https://en.wikipedia.org/wiki/Normal_distribution

[06] https://en.wikipedia.org/wiki/Standard_error

[07] Dodge, Y. (2006) *The Oxford Dictionary of Statistical Terms*, OUP. ISBN 0-19-920613-9

[08] https://en.wikipedia.org/wiki/Probable_error

Special statement: ***All the particulars provided here are as study material for the students and not for any other purpose. It has been sincerely tried to mention the concerned references wherever required.***