Semester - VI (H) / Paper DSE - 3
Harrod - Domar growth model:

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Although Harrod and Domar models of growth differ in details, they are similar in substance. Both these models stress the essential conditions of achieving and maintaining steady growth. Harrod and Domar assign a crucial role to capital accumu -lation in the process of growth.

Assumptions:

The main assumption of the Harrod -Domar models are as follows:

(1) In any period, when we start our analysis, there is a distribution of resources [e.g., capital (K) and labour (L)] which are privately owned by individuals. These individuals form the decision making. These individuals are divided into two categories: households & firms. We also have the concept of representative household & representative firm.

The households with given resources (Income) and preferences for different commodities, present & future consumption

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& work and leisure and some maximising objective has certain demand plan for different commodities.

The firms given technology and some maximising objective has supply plan of different commodities and demand for K & L.

(2) There is no government interference in the functioning of the economy.

(3) The economy is "closed economy".

(4) Exchange of commodity & resources

(5) Special assumption of only one commodity which is produced from K & L.

(6) Capital-output ratio $\left(\frac{K}{Y}\right)$ = v & Labour-output ratio $\left(\frac{L}{Y}\right)$ = a are fixed in the economy.

Harrod model:

Saving function in Harrod model is $S_t = sY_{t-1}$

where S_t is saving in period t & Y_{t-1} is output in $(t-1)^{th}$ period.

Investment function in Harrod model is $I_t = v(Y_t - Y_{t-1})$

where I_t is investment in period t & Y_t & Y_{t-1} are outputs in t & $(t-1)^{th}$ period.

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To begin with, we assume away labour market. So, for equilibrium in any period t, it is required that there should be equilibrium in capital market which requires $K_t^s = K_t^d$, where K_t^s is supply of capital in tth period & K_t^d is demand for capital in tth period. It also require equilibrium in commodity market, which require (1) $I_t = S_t$ & (2) $C_s^t = C_d^t$; where C_s^t is supply of consumption goods in tth period & C_d^t is demand for consumption goods in tth period. But the condition $C_s^t = C_d^t$ is superflous given the Walras' Law.

Let us suppose $K_t^s = K_t^d$ hold then for equilibrium

$$I_t = S_t$$

$$\text{or } v(Y_t - Y_{t-1}) = sY_{t-1}$$

$$\text{or } \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{s}{v}$$

$$\text{or } g_t = \frac{s}{v}$$

where $g_t \ (= G_w)$ is warranted rate of growth

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So, if $g_t = \frac{s}{v}$ the economy will grow in equilibrium

Since $g_t = \frac{s}{v}$

$\Rightarrow S_t = I_t$

or $K^s_{t+1} - K^s_t = K^d_{t+1} - K^d_t$

or $K^s_{t+1} = K^d_{t+1}$ $[\because K^s_t = K^d_t]$

So the stock condition in the next period hold.

Again, $I_{t+1} = S_{t+1}$

or, $v(Y_{t+1} - Y_t) = sY_t$

or, $\frac{Y_{t+1} - Y_t}{Y_t} = \frac{s}{v}$

or $g_{t+1} = \frac{s}{v}$

ensures that $K^s_{t+2} = K^d_{t+2}$

So, beginning with any period t,

if $K^s_t = K^d_t$, and then

$g_t = \frac{s}{v}$, $\forall t$

will imply that the economy will grow in equilibrium.

<u>Knife edge instability theorem:</u>

If in any period t, $K^s_t = K^d_t$

and $g_t < \frac{s}{v}$

then $\cdots g_{t+3} < g_{t+2} < g_{t+1} < g_t < \frac{s}{v}$

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And, if in any period t, $K_t^s = K_t^d$
and $g_t > \frac{s}{v}$
then $\cdots g_{t+3} > g_{t+2} > g_{t+1} > g_t > \frac{s}{v}$

By assumption, $K_t^s = K_t^d$
Then suppose the representative firm chooses $g_t < \frac{s}{v}$
Corresponding to this choice of g_t
there exist a certain Y_t and a correspon-ding maximum profit (π_t^*).
where $\pi_t^* = Y_t - r K_t^d$
Since $r = \bar{r}$ and $K_t^d = K_t^s$
π_t^* will be realised if Y_t is realised.
But $g_t < \frac{s}{v}$

or, $\frac{Y_t - Y_{t-1}}{Y_{t-1}} < \frac{s}{v}$

or, $v(Y_t - Y_{t-1}) < s Y_{t-1}$

or, $I_t < S_t$

But by Walras Law,

$$(C_t^d - C_t^s) + (S_t - I_t) + \bar{r}(K_t^d - K_t^s) \equiv 0$$

or, $(C_t^d - C_t^s) \equiv (I_t - S_t) + \bar{r}(K_t^s - K_t^d)$

But $K_t^s = K_t^d$ & $I_t - S_t < 0$

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therefore, $C^d_t - C^s_t < 0$

$\therefore C^d_t < C^s_t$

or, $C^d_t < Y_t - I_t$

or, $C^d_t + I_t < Y_t$

$\therefore$ Planned demand for output is less than planned supply of output

$\therefore Y_t$ cannot be sold

$\therefore \pi^*_t$ will not be realised.

$\therefore g_{t+1} < g_t$

So, if in any period t, $K^s_t = K^d_t$

and $g_t < \frac{s}{v}$

then ... $g_{t+3} < g_{t+2} < g_{t+1} < g_t < \frac{s}{v}$.

Similarly, it may be proved,

if in any period t, $K^s_t = K^d_t$

and $g_t > \frac{s}{v}$

then ... $g_{t+3} > g_{t+2} > g_{t+1} > g_t > \frac{s}{v}$

Dynamic equilibrium and instability

By taking into account commodity, capital and labour markets conditions for dynamic equilibrium:

(a) Equilibrium in all the relevant markets within every period

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(b) Equilibria of the different period should be interrelated

Conditions (a) & (b) involving labour market require

(1) $L^s_t = L^d_t$

& (2) $L^s_{t+1} - L^s_t = L^d_{t+1} - L^d_t$ for all t

where, L^s_t & L^s_{t+1} is supply of labour in t th & (t+1) th period respectively & L^d_t & L^d_{t+1} is demand for labour in t th & (t+1) th period respective

Labour supply has been assume to grow at a rate n.

$$\therefore \frac{L^s_{t+1} - L^s_t}{L^s_t} = n$$

$$\because L^d_{t+1} = aY_t$$

$$\& \; L^d_t = aY_{t-1}$$

$$\therefore L^d_{t+1} - L^d_t = a(Y_t - Y_{t-1})$$

Now, $L^s_{t+1} - L^s_t = L^d_{t+1} - L^d_t$ [By con (2)

or, $$\frac{L^s_{t+1} - L^s_t}{L^s_t} = \frac{L^d_{t+1} - L^d_t}{L^s_t}$$

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$$\text{or, } n = \frac{L^d_{t+1} - L^d_t}{L^d_t} \qquad [\because L^s_t = L^d_t]$$

$$\text{or, } n = \frac{a(Y_t - Y_{t-1})}{aY_{t-1}}$$

$$\text{or, } n = g_t$$

So, for dynamic equilibrium involving commodity, capital & labour markets it require

$$K^s_t = K^d_t$$

$$g_t = \frac{s}{v}$$

$$\& \; g_t = n$$

It is highly improbable that s, v and n will always have such values such that $\frac{s}{v} = n$.

Two types of instability in Harrod growth model are,

$g_t \neq \frac{s}{v}$ which is knife edge instability

& $\frac{s}{v} \neq n$ which is long-run instability

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