## **Economics Honours (Semester II)**

## **Mathematical Economics -I**

## **Constrained Optimization (Applications)**

1. Find the minimum of  $u = 2x^2 - 6y^2$  under the condition x+2y = 4. What is the value of u?

2. Find the maximum of  $u = x^2 + 3xy - 5y^2$ , given 2x+3y = 6. What is the value of u?

3. Let the utility function be  $u = q_1q_2$ . Let prices be  $p_1=1$  and  $p_2=3$ . And income M =15(i.e.

 $p_1q_1 + p_2q_2 = 15$ ). Find  $q_1$  and  $q_2$  that will maximize u.

4. Given a cost function  $C = r_1 x_1 + r_2 x_2$  and a constraint  $q = f(x_1, x_2)$ , find the first order condition for a minimum C (cost), given output (q).

5. Under the assumptions of pure competition, if total revenue is given as (R = p q), where p is the price,

q is the quantity of commodity and let C be the total cost, determine the necessary and sufficient conditions of profit maximization.

6. Assume a monopolist has respectively the following demand and average cost curves:

$$q = 400 - 20p$$
 where  $q =$  output and  $p =$  price

and

$$AC = 5 + q/20$$

Find the conditions of profit maximization. Also determine the maximum profit level, output level and price.

7. Establish the profit maximizing conditions of a discriminating monopolist and a multiplant monopolist.

8. The utility function of a consumer is given by  $u = 5 \log x_1 + 2 \log x_2$  and the budget constraint is  $4x_1 + 2x_2 = 28$ . Find out the consumer's equilibrium purchase of  $x_1$  and  $x_2$ .

9. A producer wants to minimize his cost of production given by C = 2L + 5K where L and K are inputs, subject to the production function Q=LK. Find out the optimum amount of L and K in this problem. State the 1<sup>st</sup> and 2<sup>nd</sup> order conditions of maximization of a function.

10. A utility function and corresponding budget constraint is given by U = xy and  $M = p_x x + p_y y$ . Formulate the Lagrangian function. Derive the demand function for x.

11. A monopolist has the following total revenue and total cost functions as:  $R = 30q - q^2$ ,  $C = q^3 - 15q^2$ 

+ 10q +100. Find out the profit maximizing output and the maximum profit. Derive Lerner's measure of monopoly power at the equilibrium output.

[Hint: Learner's index of monopoly power: L = (P-MC)/P]

12. What are the first and second order conditions of maximization of the utility function  $U = \alpha + \beta x + \Upsilon x^2$ , where  $\alpha > 0$ ,  $\beta > 0$ ,  $\Upsilon < 0$ .

\*N.B. All problems have been taken from the following references:

References:

Yamane, T. (1962). *Mathematics for economists: An elementary survey* (2nd ed.). Prentice hall of India Private Limited.

Question Papers of previous years.