63

TRANSPORTATION PROBLEMS

63.1 INTRODUCTION

A scooter production company produces scooters at the units situated at various places (called origins) and supplies them to the places where the depot (called destination) are situated.

Here the availability as well as requirements of the various depots are finite and constitute the limited resources.

This type of problem is known as distribution or transportation problem in which the key idea is to minimize the cost or the time of transportation.

In previous lessons we have considered a number of specific linear programming problems. Transportation problems are also linear programming problems and can be solved by simplex method but because of practical significance the transportation problems are of special interest and it is tedious to solve them through simplex method. Is there any alternative method to solve such problems?

63.2 OBJECTIVES

After completion of this lesson you will be able to:

- solve the transportation problems by

  (i) North-West corner rule;

  (ii) Lowest cost entry method; and

  (iii) Vogel's approximation method.

- test the optimality of the solution.

63.3 MATHEMATICAL FORMULATION OF TRANSPORTATION PROBLEM

Let there be three units, producing scooter, say, A_1, A_2 and A_3 from where the scooters are to be supplied to four depots say B_1, B_2, B_3 and B_4.

Let the number of scooters produced at A_1, A_2 and A_3 be a_1, a_2 and a_3 respectively and the demands at the depots be b_2, b_1, b_3 and b_4 respectively.

We assume the condition

$$a_1+a_2+a_3 = b_1+b_2+b_3+b_4$$

i.e., all scooters produced are supplied to the different depots. Let the cost of transportation of one scooter from A_1 to B_1 be c_{11}. Similarly, the cost of transportations in other casus are also shown in the figure and 63.1 Table 1.

Let out of a_1 scooters available at A_1, x_{11} be taken at B_1 depot, x_{12} be taken at B_2 depot and to other depots as well, as shown in the following figure and table 1.

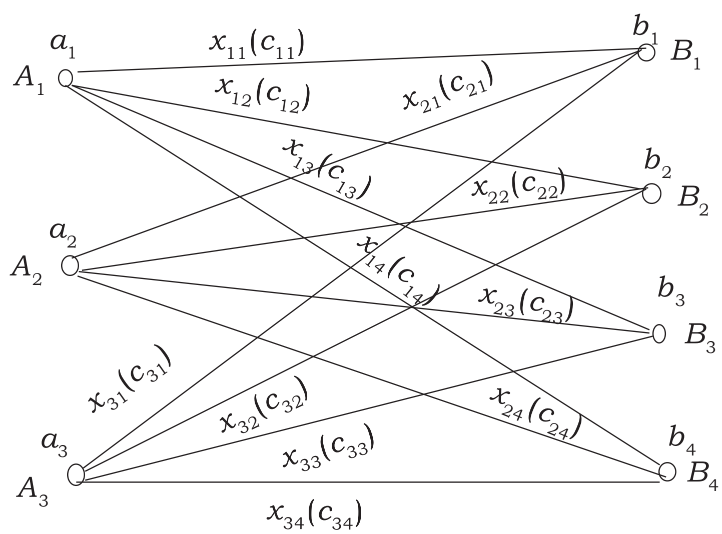


Fig 63.1

Total number of scooters to be transported form A_1 to all destination, i.e., B_1, B_2, B_3, and B_4 must be equal to a_1.

$$\therefore \qquad x_{11}+x_{12}+x_{13}+x_{14} = a_1 \qquad (1)$$

Similarly, from A_2 and A_3 the scooters transported be equal to a_2 and a_3 respectively.

$\therefore \qquad x_{21}+x_{22}+x_{23}+x_{24} = a_2 \qquad (2)$

and $\qquad x_{31}+x_{32}+x_{33}+x_{34} = a_3 \qquad (3)$

On the other hand it should be kept in mind that the total number of scooters delivered to B_1 from all units must be equal to b_1, i.e.,

$$x_{11}+x_{21}+x_{31} = b_1 \qquad (4)$$

Similarly, $\qquad x_{12}+x_{22}+x_{32} = b_2 \qquad (5)$

$$x_{13}+x_{23}+x_{33} = b_3 \qquad (6)$$

$$x_{14}+x_{24}+x_{34} = b_4 \qquad (7)$$

With the help of the above information we can construct the following table :

Table 1

Unit \ Depot	To B_1	To B_2	To B_3	To B_4	Stock
From A_1	$x_{11}(c_{11})$	$x_{12}(c_{12})$	$x_{13}(c_{13})$	$x_{14}(c_{14})$	a_1
From A_2	$x_{21}(c_{21})$	$x_{22}(c_{22})$	$x_{23}(c_{23})$	$x_{24}(c_{24})$	a_2
From A_3	$x_{31}(c_{31})$	$x_{32}(c_{32})$	$x_{33}(c_{33})$	$x_{34}(c_{34})$	a_3
Requirement	b_1	b_2	b_3	b_4	

The cost of transportation from A_i $(i=1,2,3)$ to B_j $(j=1,2,3,4)$ will be equal to

$$S = \sum_{i,j} c_{ij}\ x_{ij}\ , \qquad (8)$$

where the symbol $\sum_{i,j}$ put before $c_{ij}\ x_{ij}$ signifies that the quantities $c_{ij}\ x_{ij}$ must be summed over all $i = 1,2,3$ and all $j = 1,2,3,4$.

Thus we come across a linear programming problem given by equations (1) to (7) and a linear function (8).

We have to find the non-negative solutions of the system such that it minimizes the function (8).

Note :

We can think about a transportation problem in a general way if there are m sources (say A_1, A_2, ..., A_m) and n destinations (say B_1, B_2,....,B_n). We can use a_i to denote the quantity of goods concentrated at points A_i(i=1,2,...., m) and b_j denote the quantity of goods expected at points B_j(j=1,2,...,n). We assume the condition

$$a_1+a_2+....+a_m=b_1+b_2+....+b_n$$

implying that the total stock of goods is equal to the summed demand for it.

63.3.1 Some Definitions

The following terms are to be defined with reference to the transportation problems:

(A) Feasible Solution (F.S.)

A set of non-negative allocations $x_{ij} \geq 0$ which satisfies the row and column restrictions is known as feasible solution.

(B) Basic Feasible Solution (B.F.S.)

A feasible solution to a m-origin and n-destination problem is said to be basic feasible solution if the number of positive allocations are (m+n–1).

If the number of allocations in a basic feasible solutions are less than (m+n–1), it is called degenerate basic feasible solution (DBFS) (otherwise non-degenerate).

(C) Optimal Solution

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

63.4 SOLUTION OF THE TRANSPORTATION PROBLEM

Let us consider the numerical version of the problem stated in the introduction and the mathematical formulation of the same in the next section, as below in Table 2.

Table 2

Depot / Unit	B_1	B_2	B_3	B_4	Stock
A_1	$c_{11}=2$	$c_{12}=3$	$c_{13}=5$	$c_{14}=1$	$a_1=8$
A_2	$c_{21}=7$	$c_{22}=3$	$c_{23}=4$	$c_{24}=6$	$a_2=10$
A_3	$c_{31}=4$	$c_{32}=1$	$c_{33}=7$	$c_{34}=2$	$a_3=20$
Requirement	$b_1=6$	$b_2=8$	$b_3=9$	$b_4=15$	= =38

(All terms are in hundreds)

In order to find the solution of this transportation problem we have to follow the steps given below.

(A) Initial basic feasible solution

(B) Test for optimization.

Let us consider these steps one by one.

(A) Initial Basic Feasible Solution :

There are three different methods to obtain the initial basic feasible solution viz.

(I) North-West corner rule

(II) Lowest cost entry method

(III) Vogel's approximation method

In the light of above problem let us discuss one by one.

(I) North-West corner rule

In this method we distribute the available units in rows and column in such a way that the sum will remain the same. We have to follow the steps given below.

(a) Start allocations from north-west corner, i.e., from (1,1) position. Here min (a_1, b_1), i.e., min (8,6)=6 units. Therefore, the maximum possible units that can be allocated to this position is 6, and write it as 6(2) in the (1,1) position of the table. This completes the allocation in the first column and cross the other positions, i.e., (2,1) and (3,1) in the column. (see Table 3)

Table 3

Unit \ Depot	B_1	B_2	B_3	B_4	Stock
A_1	6(2)				8–6=2
A_2	×				10
A_3	×				20
Requirement	6–6=0	8	9	15	32

(b) After completion of step (a), come across the position (1,2). Here min (8–6,8)=2 units can be allocated to this position and write it as 2(3). This completes the allocations in the first row and cross the other positions, i.e., (1,3) and (1,4) in this row (see Table 4).

Table 4

Unit \ Depot	B_1	B_2	B_3	B_4	Stock
A_1	6(2)	2(3)	×	×	2–2=0
A_2	×				10
A_3	×				20
Requirement	0	8–2=6	9	15	30

(c) Now come to second row, here the position (2,1) is already been struck off, so consider the position (2,2). Here min (10,8–2)=6 units can be allocated to this position and write it as 6(3). This completes the allocations in second column so strike off the position (3,2) (see Table 5)

Table 5

Unit \ Depot	B_1	B_2	B_3	B_4	Stock
A_1	6(2)	2(3)	×	×	0
A_2	×	6(3)			10–6=4
A_3	×	×			20
Requirement	0	0	9	15	24

(d) Again consider the position (2,3). Here, min (10–6,9)=4 units can be allocated to this position and write it as 4(4). This completes the allocations in second row so struck off the position (2,4) (see Table 6).

Table 6

Unit \ Depot	B_1	B_2	B_3	B_4	Stock
A_1	6(2)	2(3)	×	×	0
A_2	×	6(3)	4(4)	×	0
A_3	×	×			20
Requirement	0	0	9–4=5	15	20

(e) In the third row, positions (3,1) and (3,2) are already been struck off so consider the position (3,3) and allocate it the maximum possible units, i.e., min (20,9–4)=5 units and write it as 5(7). Finally, allocate the remaining units to the position (3,4), i.e., 15 units to this position and write it as 15(2).

Keeping in mind all the allocations done in the above method complete the table as follows:

Table 7

Depot / Unit	B_1	B_2	B_3	B_4	Stock
A_1	6(2)	2(3)	×	×	8
A_2	×	6(3)	4(4)	×	10
A_3	×	×	5(7)	15(2)	20
Requirement	6	8	9	15	38

From the above table calculate the cost of transportation as $6\times2 + 2\times3 + 6\times3 + 4\times4 + 5\times7 + 15\times2$

$= 12 + 6 + 18 + 16 + 35 + 30$

$= 117$

i.e., Rs. 11700.

(II) Lowest cost entry method

(a) In this method we start with the lowest cost position. Here it is (1,4) and (3,2) positions, allocate the maximum possible units to these positions, i.e., 8 units to the position (1,4) and 8 units to position (3,2), write them as 8(1) and 8(1) respectively, then strike off the other positions in row 1 and also in column 2, since all the available units are distributed to these positions.

Table 8

Depot / Unit	B_1	B_2	B_3	B_4	Stock
A_1	×	×	×	8(1)	0
A_2		×			10
A_3		8(1)			12
Requirement	6	0	9	7	22

(b) Consider the next higher cost positions, i.e., (1,1) and (3,4) positions, but the position (1,1) is already been struck off so we can't allocate any units to this position. Now allocate the maximum possible units to position (3,4), i.e., 7 units as required by the place and write it as 7(2). Hence the allocations in the column 4 is complete, so strike off the (2,4) position.

Table 9

Unit \ Depot	B_1	B_2	B_3	B_4	Stock
A_1	×	×	×	8(1)	0
A_2		×		×	10
A_3		8(1)		7(2)	5
Requirement	6	0	9	0	15

(c) Again consider the next higher cost position, i.e., (1,2) and (2,2) positions, but these positions are already been struck off so we cannot allocate any units to these positions.

(d) Consider the next higher positions, i.e., (2,3) and (3,1) positions, allocate the maximum possible units to these positions, i.e., 9 units to position (2,3) and 5 units to position (3,1), write them as 9(4) and 5(4) respectively. In this way allocation in column 3 is complete so strike off the (3,3) position.

Table 10

Unit \ Depot	B_1	B_2	B_3	B_4	Stock
A_1	×	×	×	8(1)	0
A_2		×	9(4)	×	1
A_3	5(4)	8(1)	×	7(2)	0
Requirement	1	0	0	0	1

(e) Now only the position (2,1) remains and it automatically takes the alloation 1 to complete the total in this row, therefore, write it as 1(7).

With the help of the above facts complete the allocation table as given below.

Table 11

Unit \ Depot	B_1	B_2	B_3	B_4	Stock
A_1	×	×	×	8(1)	8
A_2	1(7)	×	9(4)	×	10
A_3	5(4)	8(1)	×	7(2)	20
Requirement	6	8	9	15	38

From the above facts, calculate the cost of transportation as

$8\times1 + 1\times7 + 9\times4 + 5\times4 + 8\times1 + 7\times2$

$= 8 + 7 + 36 + 20 + 8 + 14$

$= 93$

i.e., Rs. 9300.

(III) Vogel's approximation method

(a_1) Write the difference of minimum cost and next to minimum cost against each row in the penalty column. (This difference is known as penalty).

(a_2) Write the difference of minimum cost and next to minimum cost against each column in the penalty row. (This difference is known as penalty).

We obtain the table as given below.

Table 12

Unit \ Depot	B_1	B_2	B_3	B_4	Stocks	Penalties
A_1	(2)	(3)	(5)	(1)	8	(1)
A_2	(7)	(3)	(4)	(6)	10	(1)
A_3	(4)	(1)	(7)	(2)	20	(1)
Requirement	6	8	9	15	38	
Penalties	(2)	(2)	(1)	(1)		

↑

(b) Identify the maximum penalties. In this case it is at column one and at column two. Consider any of the two columns, (here take first column) and allocate the maximum units to the place where the cost is minimum (here the position (1,1) has minimum cost so allocate the maximum possible units, i.e., 6 units to this positon). Now write the remaining stock in row one. After removing the first column and then by repeating the step (a), we obtain as follows:

Table 13

Unit \ Depot		B_2	B_3	B_4	Stocks	Penalties	
A_1		(3)	(5)	(1)	2	(2)	←
A_2		(3)	(4)	(6)	10	(1)	
A_3		(1)	(7)	(2)	20	(1)	
Requirement		8	9	15	32		
Penalties		(2)	(1)	(1)			

↑

(c) Identify the maximum penalties. In this case it is at row one and at column two. Consider any of the two (let it be first row) and allocate the maximum possible units to the place where the cost is minimum (here the position (1,4) has minimum cost so allocate the maximum possible units, i.e., 2 units to this position). Now write the remaining stock in column four. After removing the first row and by repeating the step(a), we obtain table 14 as given below.

Table 14

Depot / Unit		B_2	B_3	B_4	Stocks	Penalties
A_2		(3)	(4)	(6)	10	(1)
A_3		(1)	(7)	(2)	20	(1)
Requirement		8	9	13	30	
Penalties		(2)	(3)	(4) ↑		

(d) Identify the maximum penalties. In this case it is at column four. Now allocate the maximum possible units to the minimum cost position (here it is at (3,4) position and allocate maximum possible units, i.e., 13 to this positon). Now write the remaining stock in row three. After removing the fourth column and then by repeating the step (a) we obtain table 15 as given below.

Table 15

Depot / Unit		B_2	B_3		Stock	Penalties
A_2		(3)	(4)		10	(1)
A_3		(1)	(7)		7	(6)
Requirement		8	9			
Penalties		(2)	(3)			

(e) Identify the maximum penalties. In this case it is at row three. Now allocate the maximum possible units to the minimum cost position (here it is at (3,2) position and allocate maximum possible units, i.e., 7 to this position). Now in order to complete the sum, (2,2) position will take 1 unit and (2,3) position will be allocated 9 units.

This completes the allocation and with the help of the above informations draw table 16 as under.

Table 16

Unit \ Depot	B_1	B_2	B_3	B_4	Stock
A_1	6(2)			2(1)	8
A_2		1(3)	9(4)		10
A_3		7(1)		13(2)	20
Requirement	6	8	9	15	38

From the above facts calculate the cost of transportation as

$6 \times 2 + 2 \times 1 + 1 \times 3 + 9 \times 4 + 7 \times 1 + 13 \times 2$

$= 12 + 2 + 3 + 36 + 7 + 26$

$= 86$

i.e., Rs. 8600.

Note:

After calculating the cost of transportation by the above three methods, one thing is clear that Vogel's approximation method gives an initial basic feasible solution which is much closer to the optimal solution than the other two methods. It is always worth while to spend some time finding a "good" initial solution because it can considerably reduce the total number of iterations required to reach an optimal solution.

INTEXT QUESTIONS 63.1

1. Determine an initial basic feasible solution to the following transportaion problem using north-west corner rule, where O_i and D_j represent i^{th} origin and j^{th} destination respectively.

Table 17

	D_1	D_2	D_3	D_4	Supply
O_1	10	8	5	9	14
O_2	12	13	6	11	16
O_3	8	7	10	6	5
Demand	6	10	15	4	35

2. Find the initial basic feasible solution to the following transportation problem by lowest cost entry method.

Table 18

	D_1	D_2	D_3	D_4	Supply
O_1	3	7	5	5	38
O_2	5	5	3	4	19
O_3	2	4	4	5	16
O_4	4	9	4	6	23
Demand	25	29	21	21	96

3. Determine an initial basic feasible solution using Vogel's method by considering the following transportation problem.

Table 19

	D_1	D_2	D_3	D_4	Supply
O_1	21	16	15	3	11
O_2	17	18	14	23	13
O_3	32	27	18	41	19
Demand	6	10	12	15	43

(B) Test for Optimization :

In part (A) of this section we have learnt how to obtain an initial basic feasible solution. Solutions so obtained may be optimal or may not be optimal, so it becomes essential for us to test for optimization.

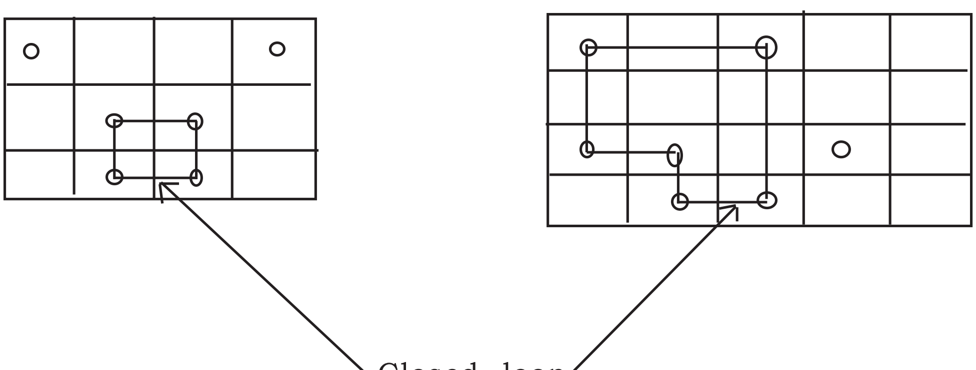
For this purpose we first define non-degenerate basic feasible solution.

Definition: A basic feasible solution of an ($m \times n$) transportation problem is said to be **non-degenerate** if it has following two properties :

(a) Initial basic feasible solution must contain exactly $m+n-1$ number of individual allocations.

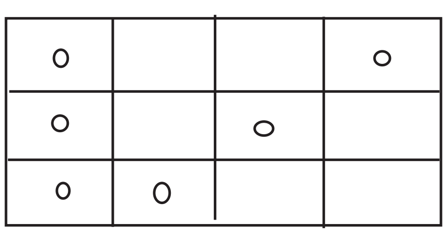
(b) These allocations must be in independent positions.

Independent positions of a set of allocations means that it is always impossible to form any closed loop through these allocations. See fig. given below.



Closed loop

Non - independent positions



Independent positons

The following theorem is also helpful in testing the optimality.

Theorem: If we have a B.F.S. consisting of $m+n-1$ independent positive allocations and a set of arbitrary number u_i and v_j ($i=1,2,...m$; $j=1,2,...n$) such that $c_{rs} = u_r+v_s$ for all occupied cells (r,s) then the evaluation d_{ij} corresponding to each empty cell (i, j) is given by

$$d_{ij} = c_{ij} - (u_i+v_j)$$

Algorithm for optimality test :

In order to test for optimality we should follow the procedure as given below:

Step 1: Start with B.F.S. consisting of $m+n-1$ allocations in independent positions.

Step 2: Determine a set of $m+n$ numbers

u_i ($i=1,2,....m$) and v_j ($j=1,2,...n$)

such that for each occupied cells (r,s)

$$c_{rs} = u_r+v_s$$

Step 3: Calculate cell evaluations (unit cost difference)

d_j for each empty cell (i,j) by using the formula

$$d_{ij} = c_{ij} - (u_i+v_j)$$

Step 4: Examine the matrix of cell evaluation d_{ij} for negative entries and conclude that

(i) If all $d_{ij} > 0 \Rightarrow$ Solution is optimal and unique.

(ii)If all $d_{ij} \geq 0 \Rightarrow$ At least one $d_{ij} = 0$

$\Rightarrow$ Solution is optimal and alternate solution also exists.

(iii) If at least one $d_{ij} < 0 \Rightarrow$ Solution is not optimal.

If it is so, further improvement is required by repeating the above process. See step 5 and onwards.

Step 5: (i) See the most negative cell in the matrix [d_{ij}].

(ii) Allocate θ to this empty cell in the final allocation table. Subtract and add the amount of this allocation to other corners of the loop in order to restore feasibility.

(iii) The value of θ, in general is obtained by equating to zero the minimum of the allocations containing $-\theta$

(not + θ) only at the corners of the closed loop.

(iv) Substitute the value of θ and find a fresh allocation table.

Step 6: Again, apply the above test for optimality till you find all $d_{ij} \geq 0$

Computational demonstration for optimality test

Consider the initial basic feasible solution as obtained by Vogel's approximation method in section (A) of this article [Table (16)].

Step 1: (i) In this table number of allocations = 3+4–1=6.

(ii) All the positions of allocations are independent.

Step 2: Determine a set of $(m+n)$, i.e., (3+4) numbers u_1, u_2, u_3, and v_1, v_2, v_3, and v_4. for each occupied cells.

For this consider the row or column in which the allocations are maximum (here, let us take first row). Now, take u_1 as an arbitrary constant (say zero) then by using $c_{ij} = u_i+v_j$ try to find all u_i and v_j as

Table 20

	B_1	B_2	B_3	B_4	u_i
A_1	2			1	0
A_2		3	4		3
A_3		1		2	1
v_j	2	0	1	1	

$c_{11} = 2 = u_1+v_1 = 0+v_1 \Rightarrow v_1 = 2$

then $c_{14} = 1 = u_1+v_4 = 0+v_4 \Rightarrow v_4 = 1$

then $c_{34} = 2 = u_3+v_4 = u_3+1 \Rightarrow u_3 = 1$

then $c_{32} = 1 = u_3+v_2 = 1+v_2 \Rightarrow v_2 = 0$

then $c_{22} = 3 = u_2+v_2 = u_2+0 \Rightarrow u_2 = 3$

then $c_{23} = 4 = u_2+v_3 = 3+v_3 \Rightarrow v_3 = 1$

Thus $u_1 = 0$, $u_2 = 3$, $u_3 = 1$ and $v_1 = 2$, $v_2 = 0$, $v_3 = 1$ and $v_4 = 1$.

Step 3: Cost matrix for the empty positons

i.e., $[\ c_{ij}\ ] = \begin{bmatrix} \bullet & 3 & 5 & \bullet \\ 7 & \bullet & \bullet & 6 \\ 4 & \bullet & 7 & \bullet \end{bmatrix}$

Matrix $[\ u_i + v_j\ ]$ for empty positions

i.e., $[\ u_i + v_j\ ] = \begin{bmatrix} \bullet & 0 & 1 & \bullet \\ 5 & \bullet & \bullet & 4 \\ 3 & \bullet & 2 & \bullet \end{bmatrix}$

$\therefore\ d_{ij}=[c_{ij}]-[u_i+v_j] = \begin{bmatrix} \bullet & 3 & 4 & \bullet \\ 2 & \bullet & \bullet & 2 \\ 1 & \bullet & 5 & \bullet \end{bmatrix}$

Step 4: Here all $d_{ij} > 0 \Rightarrow$ Solution obtained by Vogel's approximation method is an optimal solution

Example A: For the transportation problem

Table 21

Warehouse → / Factory	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

the initial basic feasible solution obtained by Vogel's approximation method is given below.

Table 22

	W_1	W_2	W_3	W_4	Available
F_1	5(19)			2(10)	7
F_2			7(40)	2(60)	9
F_3		8(8)		10(20)	18
Requirement	5	8	7	14	

Test this for optimality.

Solution :

Step 1: Number of allocations = 3+4–1=6 and they are in independent positon.

Step 2:

					u_i
	(19)			(10)	10
			(40)	(60)	60
		(8)		(20)	20
v_j	9	–12	–20	0	

Step 3:

$[c_{ij}]$ =

	30	50	
70	30		
40		70	

$[u_i+v_j]$=

					u_i
		–2	–10		10
	69	48			60
	29		0		20
v_j	9	–12	–20	0	

$d_{ij}=[c_{ij}]-[u_i+v_j]=$

	32	60	
1	–18		
11		70	

Step 4: Since one $d_{ij} < 0$, therefore, the solution is not optimal. See step 5 and onwards for to find new allocations and test of optimality

Step 5: (ii)

					Available
	5(19)			2(10)	7
		$+\theta$	7(40)	2(60) $-\theta$	9
		8(8) $-\theta$		10(20) $+\theta$	18
Required	5	8	7	14	

(iii) $\min[8-\theta, 2-\theta] = 0$

i.e., $2-\theta = 0$

i.e., $\theta = 2$

					Available
	5(19)			2(10)	7
$\therefore$		2(30)	7(40)		9
		6(8)		12(20)	18
Required	5	8	7	14	

This improved basic feasible solution gives the cost for this solution as

5(19)+2(10)+2(30)+7(40)+6(8)+12(20) = Rs.743.

Step 6: Test this improved solution for optimality by repeating steps 1,2,3 and 4. In each step, following matrices are obtained:

Matrix $[c_{ij}]$ for empty cells

•	(30)	(50)	•
70	•	•	(60)
(40)	•	(70)	•

Matrix for u_i and v_j

					u_i
	•(19)			•(10)	−10
		•(30)	•(40)		22
		•(8)		•(20)	0
v_j	29	8	18	20	

Matrix $[u_i+v_j]$ for empty cells

•	−2	8	•
51	•	•	42
29	•	18	•

Matrix for $d_{ij}=[c_{ij}]-[u_i+v_j]$ for empty cells

•	32	42	•
19	•	•	18
11	•	52	•

Since all $d_{ij} > 0$, therefore, the solution given as improved basic feasible solution is an optimal solution with minimum cost = Rs.743.

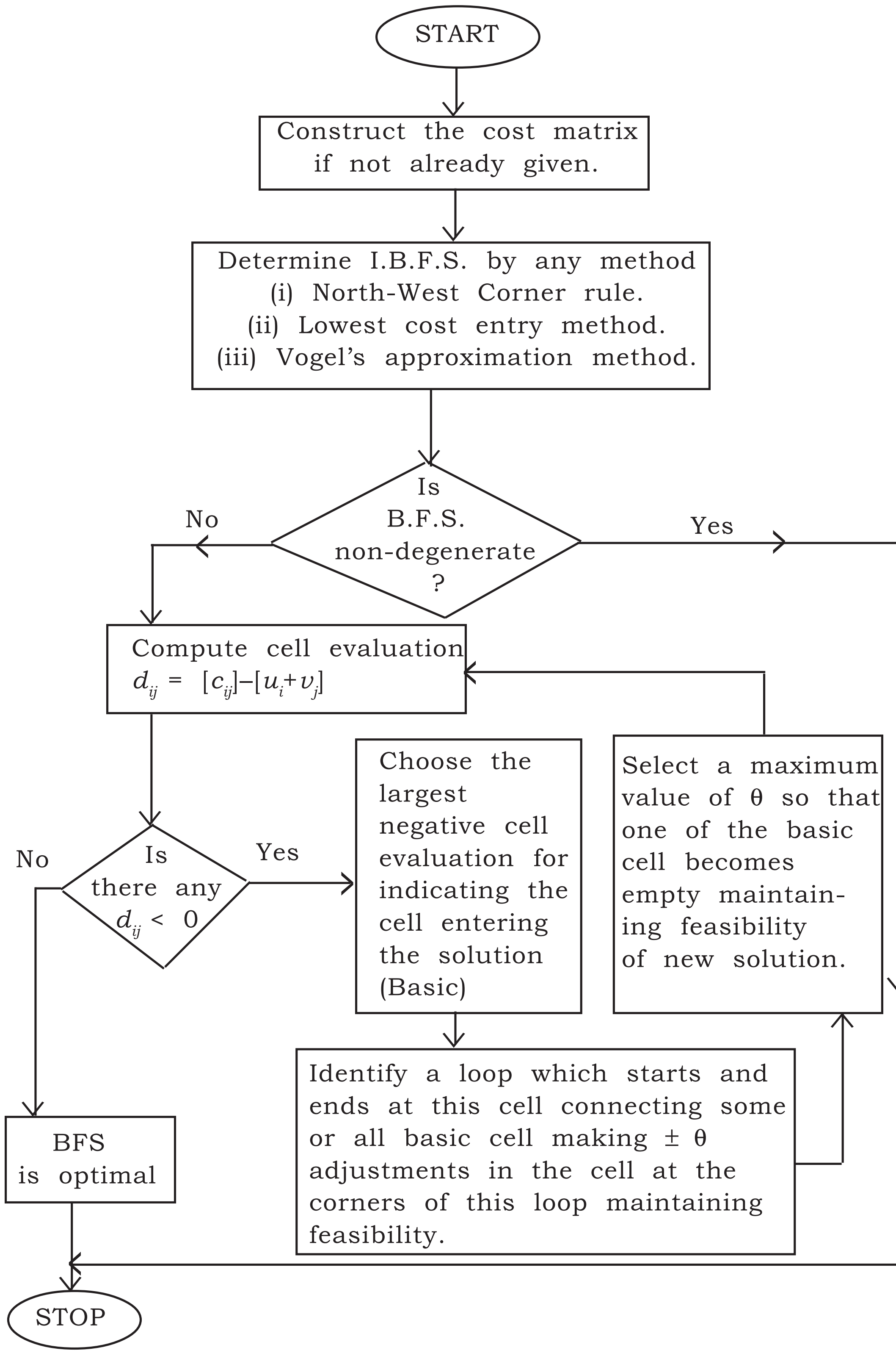
INTEXT QUESTIONS 63.2

1. Test for optimality of the solution obtained in question 1 of Intext Questions 63.1. Give the final allocation table and the minimum cost for the problem.

2. Test for optimality of the solution obtained in question 2 of Intext Questions 63.1. Give the final allocation table and the minimum cost for the problem.

3. Test for optimality of the solution obtained in question 3 of Intext Questions 63.1. Give the final allocation table and the minimum cost for the problem.

WHAT YOU HAVE LEARNT



TERMINAL QUESTIONS

1. Solve the following transportation problem :

	B_1	B_2	B_3	B_4	Stock
A_1	1	2	1	4	30
A_2	3	3	2	1	50
A_3	4	2	5	9	20
Requirement	20	40	30	10	100

Also test it for optimality.

2. Solve the following transportation problem :

	B_1	B_2	B_3	B_4	Stock
A_1	21	16	25	13	11
A_2	17	18	14	23	13
A_3	32	17	18	41	19
Requirement	6	10	12	15	43

Also test it for optimality.

ANSWERS TO INTEXT QUESTIONS

63.1

1.

	D_1	D_2	D_3	D_4	Supply
O_1	6(10)	8(8)			14
O_2		2(13)	14(6)		16
O_3			1(10)	4(6)	5
Demand	6	10	15	4	35

Cost = Rs.268

2.

	D_1	D_2	D_3	D_4	Supply
O_1	9(3)	8(7)		21(5)	38
O_2			19(3)		19
O_3	16(2)				16
O_4		21(9)	2(4)		23
Demand	25	29	21	21	96

Cost = Rs.474

3.

	D_1	D_2	D_3	D_4	Supply
O_1				11(3)	11
O_2	6(17)	3(18)		4(23)	13
O_3		7(27)	12(18)		19
Demand	6	10	12	15	43

Cost = Rs.686

63.2

1. Find allocation table is

	D_1	D_2	D_3	D_4	Supply
O_1	4(10)	10(8)			14
O_2	1(12)		15(6)		16
O_3	1(8)			4(6)	5
Demand	6	10	15	4	35

$$\text{Minimum cost} = (4\times10)+(10\times8)+(1\times12)+(15\times6)+(1\times8)+(4\times6)$$

$$= 40+80+12+90+8+24$$

$$= \text{Rs.}254$$

2. Final allocation table

	D_1	D_2	D_3	D_4	Supply
O_1	23(3)			15(5)	38
O_2		13(5)		6(4)	19
O_3		16(4)			16
O_4	2(4)		21(4)		23
Demand	25	29	21	21	96

$$\text{minimum cost} = (23\times3)+(15\times5)+(13\times5)+(6\times4)+(16\times4)+(2\times4)+(21\times4)$$

$$= \text{Rs.}389$$

Alternate solution also exists.

(**Note:** Though the problem is tedious one but for practice it is a good exercise)

3. Final allocation table is

	D_1	D_2	D_3	D_4	Supply
O_1				11(3)	11
O_2	6(17)	3(18)		4(23)	13
O_3		7(27)	12(18)		19
Demand	6	10	12	15	43

$$\text{minimum cost} = (11\times3)+(6\times17)+(3\times18)+(4\times23)+(7\times27)+(12\times18)$$
$$= 33+102+54+92+189+216$$
$$= \text{Rs.}686$$

ANSWERS TO TERMINAL QUESTIONS

1. Rs. 180
2. Rs. 711