e-Study material for the students of B.Sc (Hons.) in Physics, Sem-VI
Paper: Condensed matter Physics (CC-14)
Topic: Dielectric Properties of matter ; Class No: 05 ; Date: 27th April '20
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Class No: 05 Plasma Oscillations, Plasma frequency, Plasmons, TO modes.

The dielectric function $\epsilon(\omega, \vec{K})$ of the electron gas, with its strong dependence on frequency and wave vector, has significant consequences for the physical properties of solids.

In one limit, $\epsilon(\omega, 0)$ describes the collective excitations of the Fermi sea - the volume and surface plasmons.

In another limit, $\epsilon(0, \vec{K})$ describes the electrostatic screening of the electron-electron, electron-lattice, and electron-impurity interactions in crystals.

Plasma optics

In the long wavelength range, the dielectric function $\epsilon(\omega, 0)$ or $\epsilon(\omega)$ of an electron gas is obtained from the equation of motion of a free electron in an electric field:

$$m\frac{d^2x}{dt^2} = -eE \quad \text{——— (1)}$$

If x and E have the time dependence $e^{-i\omega t}$ then,

$$-\omega^2 m x = -eE \quad ; \; x = eE/m\omega^2 \quad \text{——— (2)}$$

The dipole moment of one electron is, $-ex = -e^2E/m\omega^2$, and the polarization defined as the dipole moment per unit volume is,

$$P = -nex = -\frac{ne^2}{m\omega^2}E \quad \text{——— (3)}$$

Where, n is the electron concentration.

The dielectric function at frequency ω is,

$$\epsilon(\omega) = \frac{D(\omega)}{E(\omega)} = 1 + 4\pi\frac{P(\omega)}{E(\omega)} \quad \text{(In CGS)}$$

$$\text{or, } \epsilon(\omega) = \frac{D(\omega)}{\epsilon_0 E(\omega)} = 1 + \frac{P(\omega)}{\epsilon_0 E(\omega)} \quad \text{——— (4)}$$

The dielectric function of the free electron gas follows from (3) and (4), $\epsilon(\omega) = 1 - \dfrac{ne^2}{\epsilon_0 m\omega^2}$ ——— (5)

The plasma frequency ω_p is defined by the relation,

$$\omega_p^2 \equiv \frac{ne^2}{\epsilon_0 m} \quad —(6)$$

A plasma is a medium with equal concentration of positive and negative charges, of which at least one charge type is mobile. In a solid, the negative charges of the conduction electrons are balanced by an equal concentration of positive charge of the ion cores. We write the dielectric function(5) as,

$$\epsilon(\omega) = 1 - \omega_p^2/\omega^2 \quad —(7)$$

If the positive ion core background has a dielectric constant labeled $\epsilon(\infty)$ essentially constant upto frequencies well above ω_p, then (5) becomes,

$$\epsilon(\omega) = \epsilon(\infty) - \frac{ne^2}{m\omega^2} = \epsilon(\infty)\left[1 - \tilde{\omega}_p^2/\omega^2\right] —(8)$$

where, $\tilde{\omega}_p$ is defined as,

$$\tilde{\omega}_p^2 = \frac{ne^2}{\epsilon(\infty) m} \quad —(9)$$

Note that, $\epsilon(\omega) = 0$ at $\omega = \tilde{\omega}_p$.

Dispersion relation for EM-Waves :-

In a non-magnetic isotropic medium the EM wave equation is,

$$\mu_0 \partial^2\vec{D}/\partial t^2 = \nabla^2\vec{E} \quad —(10)$$

We look for a solution with $\vec{E} \propto \exp(-i\omega t)\exp(i\vec{K}\cdot\vec{r})$ and $\vec{D} = \epsilon(\omega, \vec{K})\vec{E}$, then we have the dispersion relation for EM-waves,

$$\epsilon(\omega, \vec{K})\epsilon_0\mu_0\omega^2 = K^2 \quad —(11)$$

This relation tells us a great deal. Consider,

1) ϵ real and >0. For ω real, K is real and a transverse EM wave propagates with the phase velocity $c/\sqrt{\epsilon}$.
2) ϵ real and <0. For ω real, K is imaginary and the wave is damped with a characteristic length $1/|\vec{K}|$.

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3) ϵ Complex, For ω real, $\vec{k}$ is complex and the waves are damped in space.

4) $\epsilon = \infty$. This means the system has a finite response in the absence of an applied force, thus the poles of $\epsilon(\omega, \vec{k})$ define the frequencies of the free oscillations of the medium.

5) $\epsilon = 0$. We shall see that longitudinally polarized waves are possible only at the zeros of ϵ.

Transverse optical modes in a plasma:-

The dispersion relation becomes, from (8),

$$\epsilon(\omega)\omega^2 = \epsilon(\infty)[\omega^2 - \tilde{\omega}_p^2] = c^2K^2 \quad \text{—(12)}$$

(using eqn (11) and $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$)

For $\omega < \tilde{\omega}_p$ we have, $K^2 < 0$, so that K is imaginary. The solutions of the wave equation are of the form $e^{-|K|x}$ in the frequency region $0 < \omega \leq \tilde{\omega}_p$.

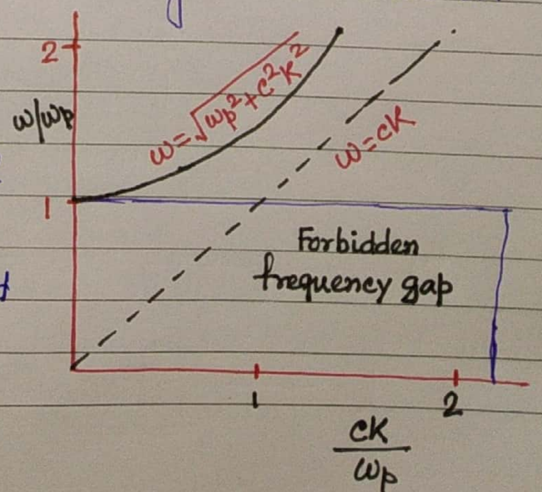
Waves incident on the medium in this frequency region do not propagate, but will be totally reflected.

An electron gas is transparent when $\omega > \tilde{\omega}_p$, for here the dielectric function is positive real. The dispersion relation in this region may be written as,

From (12), $$\omega^2 = \tilde{\omega}_p^2 + \frac{c^2K^2}{\epsilon(\infty)} \quad \text{—— (13)}$$

This describes transverse electromagnetic waves in a plasma.

Fig:- Dispersion relation for transverse electromagnetic waves in a plasma. The group velocity $v_g = d\omega/dK$ is the slope of the dispersion curve. Although the dielectric function is between zero and one, the group velocity is less than the velocity of light in vacuum.



Plasmons

A plasma oscillation in a metal is a collective longitudinal excitation of the conduction electron gas. A plasmon is a quantum of a plasma oscillation, we may excite a plasmon by passing an electron through a thin metallic film or by reflecting an electron or a photon from film. The charge of the electron couples with the electrostatic field fluctuations of the plasma oscillations. The reflected or transmitted electron will show an energy loss equal to integral multiples of the plasmon energy.

Reference

Introduction to Solid State Physics ; C Kittel