Economics Honours (Semester II)

Mathematical Economics -I

Rules of Differentiation

***Constant Rule: If y = f(x) = a where ‘a’ is a constant, then derivative of y with respect to x is: dy/dx = 0.**

Example: Suppose y =100, dy/dx = 0.

***Power Rule: If $y = f(x) = ax^n$, where a is a constant and n is a non-zero integer, then derivative of y with respect to x is: $dy/dx = nax^{n-1}$**

Example: 1. If $y=3x^2$, then $dy/dx= 2x\ 3x^{2-1}=6x^1 = 6x$.

2. If $y = x^{1/2}$, then $dy/dx = ½\ x^{-1/2}$

***Addition Rule: If y = f(x) can be written as sum of two functions, i.e., y = u(x) +v(x), then derivative of y with respect to x is:**

dy/dx = f’(x) = u’(x) + v’(x) = du/dx + dv/dx.

Example: If $y = x^3 + 5x^2 + 2x + 10$, $dy/dx = 3x^2 + 10x + 2$

***Subtraction Rule: If y = f(x) can be written as difference of two functions, i.e. y= u(x) - v(x), then derivative of y with respect to x is:**

dy/dx = f’(x) = u’(x) - v’(x) = du/dx - dv/dx.

Example: If $y = 3x^2 – 4x^3$, then $dy/dx = 6x – 12x^2$

***Product Rule: If y = f(x) can be written as the product of two functions u(x) and v(x), i.e. y = u(x).v(x), then derivative of y with respect to x is:**

dy/dx = u(x).v’(x) + v(x).u’(x) = u(x).dv/dx + v(x).du/dx

= first function x derivative of the second + second function x derivative of the first.

Example: Let $y= x^2(1+x)$, then $u(x) = x^2$ and $v(x) = (1+x)$.

Here $du/dx = d(x^2)/dx = 2x$ and $dv/dx = d(1+x)/dx = 1$

Hence $dy/dx = x^2 .d(1+x)/dx + (1+x)\, d(x^2)/dx$

$$= x^2 . 1 + (1+x). 2x$$

$$= x^2 + 2x + 2x^2$$

$$= 3x^2+2x.$$

***Quotient Rule: If y = f(x) can be written as the division of two functions u(x) and v(x), i.e. y = v(x)/u(x), then derivative of y with respect to x is:**

$dy/dx = [u(x).v'(x) - v(x).u'(x)]/ [u(x)]^2 = (u.dv/dx – v.du/dx)/u^2 =$

(second function x derivative of the first – first function x derivative of the second)/(square of the second function)

Example: Let $y = x/(1+x^2)$, then $v(x) = x$ and $u(x) = (1+x^2)$,

Hence $dy/dx = \dfrac{(1+x^2).1 - 2\, x.x}{(1+ x^2)^2} = \dfrac{1 + x^2 – 2x^2}{(1+ x^2)^2} = \dfrac{1- x^2}{(1+ x^2)^2}$

***Chain Rule: If a function y = f(x) can be written as a composite function y = g(u) where u = h(x), then the derivative of y with respect to x is equal to product of the derivative of y with respect to u and the derivative of u with respect to x, i.e. if y = g(u) and u =h(x), then dy/dx = g'(u).h'(x) = dy/du.du/dx.**

Example: Let $y = (1+x^2)^{1/2}$, Suppose $y = g(u) = u^{1/2}$ where $u = h(x) =1+x^2$, then we have $dy/du = ½ u^{-1/2}$ & $du/dx = 2x$, Hence $dy/dx = dy/du . du/dx = (½ u^{-1/2}).2x$. Inserting $u=1+x^2$, here we get, $dy/dx = ½ (1+x)^{-1/2}.2x = (x)/(1+x)^2$.

***The Logarithm Rule: If y=f(x) = In(x), then the derivative of the logarithm function is the reciprocal function, i.e. dy/dx = 1/x. Here to obtain the derivative of y = In(x), let us write y = In(u) where u = f(x), then by Chain**

Rule we obtain dy/dx = (dy/du)(du/dx). This gives us the generalized logarithm rule.

***The Generalized Logarithm Rule: If y =In f(x), then the derivative of y with respect to x is equal to: dy/dx = f'(x)/f(x)**

[Please note: 'In' means log]

***A Special case**: If y = a^x, then In y = x In a, which gives

d (In y)/dx = In a [where In (a) is a constant].

But according to the generalized logarithm rule

d (In y)/dx = 1/y. dy/dx.

Therefore, 1/y. dy/dx = In (a)

i.e. dy/dx = y. In (a)

i.e. dy/dx = a^x. In (a)

Thus the general rule if y = a^x, then derivative of y with respect to x is:

dy/dx = a^x. In (a)

***Exponential Rule: The derivative of the exponential function is the exponential function itself. Hence if the function is y = f(x) = e^x, then derivative of y with respect to x is: dy/dx = e^x.**

Example: Let y = e^{ax}. Here let us suppose y = e^u and u = ax, Then by Chain Rule and Exponential Rule we get, dy/dx = (dy/du)(du/dx)

Here dy/du = e^u & du/dx = a, Therefore dy/dx = e^u. a. Substituting the u = ax, we get dy/dx = ae^{ax}.

*N.B. Explanations and all theoretical disposition have been accessed from the following references

References

1. Birchenhall, Chris and Grout, Paul. *Mathematics for Modern Economics*, (1987), Heritage Publishers, New Delhi.