Capacitor

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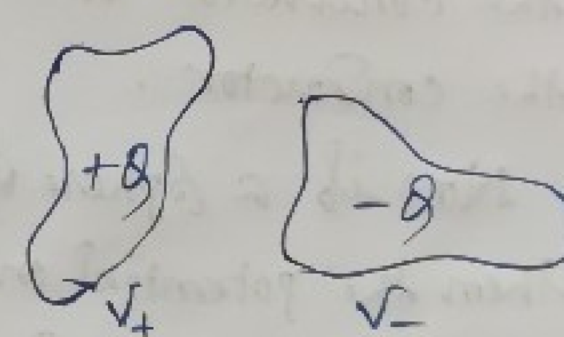
A capacitor is a device that can store electrostatic energy. In actual practice, a capacitor is consisting of two conductors separated by an insulating or dielectric medium (including air) and carring equal and opposite charge.

Suppose we have two conductors and we put $+Q$ charge on one and $-Q$ on the other. The conductor with positive charge is called positive plate and other is called negative plate. The charge on the positive plate is called the charge on the capacitor, and the potential difference between the plates is called potential of the capacitor.

From the fig, potential of the capacitor is given by



$$V = V_+ - V_-$$

For a given capacitor

$$Q \propto V$$

or. $$Q = CV$$

The proportionality constant C is called the capacitance of the capacitor which is defined as the amount of charge to increase the unit potential.

Capacitance is a purely geometrical quantity, determined by sizes, shapes and separation of the two conductors. In SI capacitance is measured in farad (F); a farad is a Coulomb/volt.

Capacitance of an isolated conductor :

We shall occasionally hear someone ~~same~~ speak of the capacitance of a single conductor. In this case the 'second conductor' with the negative charge, is an imaginary spherical shell of infinite radius surrounding the one conductor. It contributes nothing to the field. The charge on an isolated conductor Q can be written as

$$Q = CV$$

where V is the potential on the surface of the conductor (actually V is the potential difference on the surface of the conductor w.r.t infinity) and C is capacitance of the conductor.

Now if a sphere of radius 'a' containing charge 'Q' then the potential on the surface is

$$V = \frac{Q}{4\pi\epsilon_0 a}$$

$$\therefore Q/V = 4\pi\epsilon_0 a$$

So, capacitance of the spherical conductor $C = 4\pi\epsilon_0 a$

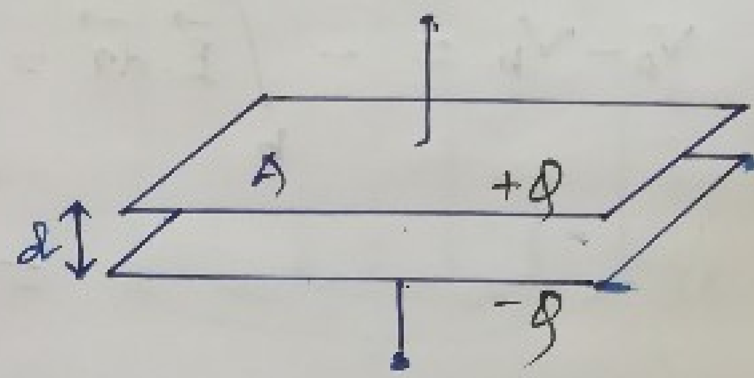
If the sphere is in a dielectric medium of relative permitivity K then

$$C = 4\pi K\epsilon_0 a$$

In CGS $4\pi\epsilon_0$ is to be replaced by unity.

Parallel plate capacitor :-

A parallel plate capacitor consist of two plates having equal area A (resonably large) separated by a small distance d. Let the charge +Q is on one plate and −Q on the other. The field E is nearly uniform in the region between the plates and is given by



$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

(σ is surface charge density).

The direction of the field is from (+)ve plate to (−)ve plate.

If V be the potential difference between the plates

then $V = Ed$

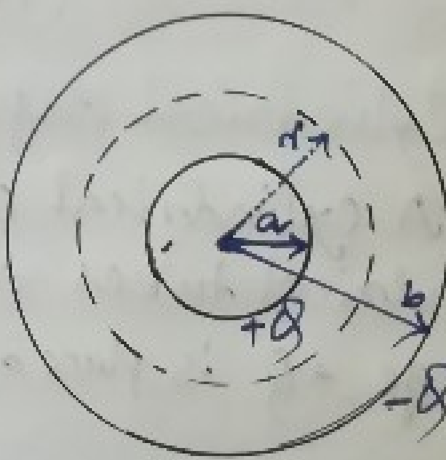
$= \left(\frac{Q}{\epsilon_0 A}\right) d$

∴ Capacitance of the capacitor is $C = \frac{Q}{V}$

$= \frac{\epsilon_0 A}{d}$

Spherical capacitor :-

A spherical capacitor consist of two concentric thin conducting spherical shells of radius a and b (where a<b). A charge +Q is placed on the inner shell and −Q on the outer one. Now to find the electric field in between the two shells let us consider a Gaussian surface of radius r. So from the Gauss's law of electrostatics



$$\oint\!\!\oint \vec{E}\cdot d\vec{s} = Q/\epsilon_0$$

$$E \cdot 4\pi r^2 = Q/\epsilon_0$$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

So, the potential difference between two shells is

$$V_a - V_b = -\int_b^a \vec{E}\cdot d\vec{r} = -\frac{Q}{4\pi\epsilon_0}\int_b^a \frac{1}{r^2}\,dr$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{b-a}{ab}\right)$$

So, the capacitance of the spherical capacitor is

$$C = \frac{Q}{V}$$

$$= 4\pi\epsilon_0\left(\frac{ab}{b-a}\right)$$

Special case :- The capacitance of an isolated spherical conductor of radius R can be found out by putting $a = R$ and $b = \infty$

$$\therefore C = 4\pi\epsilon_0\left(\frac{a}{1 - a/b}\right)$$

$$= 4\pi\epsilon_0 \frac{R}{(1 - R/\infty)} = 4\pi\epsilon_0 R.$$

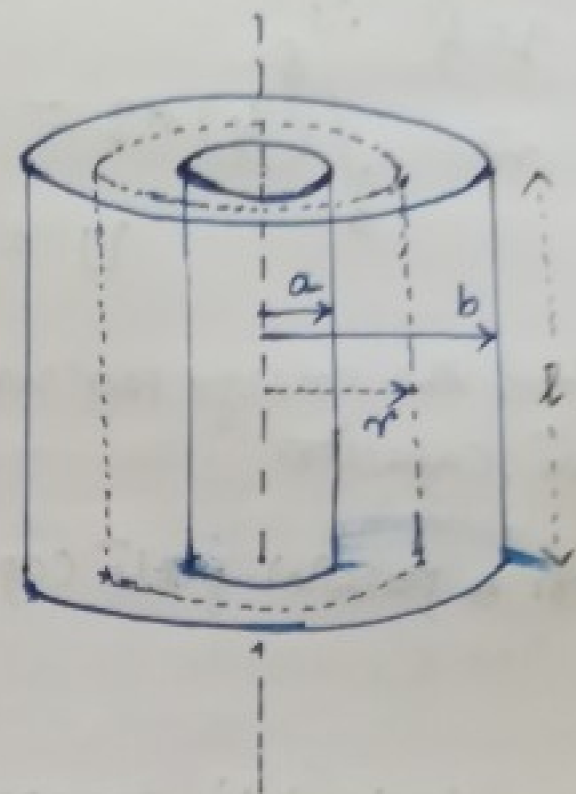
Cylindrical capacitor :-

A cylindrical capacitor consist of two coaxial metal cylindrical tubes of radii a and b (where $a < b$). A Charge $+Q$ is placed on the inner tube and $-Q$ on the outer one. Now to find the electric field in between two metallic tubes let us consider a Gaussian surface of radius r. So from Gauss's law of electrostatics

$$\oint\!\!\oint \vec{E}\cdot d\vec{s} = Q/\epsilon_0$$

$$E(2\pi r l) = Q/\epsilon_0$$

$$E = \frac{Q}{2\pi\epsilon_0 l r}$$



∴ Potential difference between inner and outer shell is

$$V_a - V_b = -\frac{Q}{2\pi\epsilon_0 l}\int_b^a \frac{1}{r}\,dr$$

$$= -\frac{Q}{2\pi\epsilon_0 l}\ln(a/b)$$

$$V = \frac{Q}{2\pi\epsilon_0 l}\ln(b/a)$$

$$\therefore C = Q/V = 2\pi\epsilon_0 \frac{l}{\ln(b/a)}$$

Energy stored in a capacitor :-

The energy of a charged capacitor is the amount of work done in charging it. (To charge up a capacitor we have to remove electrons from positive plate and carry them to negative plate. In doing so we fight against the electric field). Suppose at some intermediate stage in the process of charging let 'q' be the charge on the positive plate. So potential difference between the plates is q/c. Now the work done to transfer dq more charge from (-)ve plate to (+)ve plate is

$dw = (q/c)\, dq$.

So, total work done to charge the capacitor from $q=0$ to $q=Q$ is

$$w = \int_0^Q (q/c)\, dq = \frac{1}{2} Q^2/c$$

$$U = \frac{1}{2} cV^2 \quad [\because Q = cV]$$

* Find the energy per unit volume stored in a parallel plate capacitor.

For a parallel plate capacitor $c = \frac{\epsilon_0 A}{d}$

and $V = Ed$.

Now electrostatic energy $U = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2$

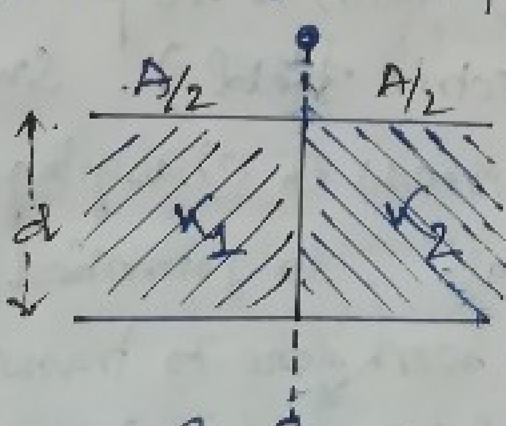
$$= \frac{1}{2}\epsilon_0 E^2 (Ad).$$

$\because$ Ad is the volume of the capacitor

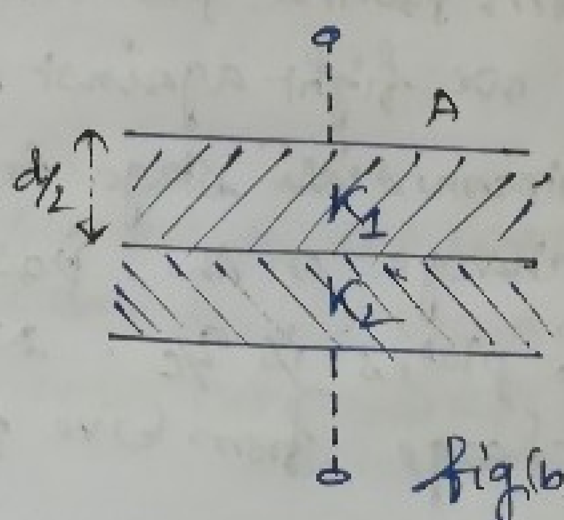
$\therefore$ Energy stored per unit volume

$$u = \frac{1}{2}\epsilon_0 E^2$$

* A parallel plate capacitor is filled with two dielectrics of same dimensions but different dielectric constants K_1 and K_2 respectively as shown in fig(a) and (b). Calculate its capacitance.



fig(a)



fig(b)

(i) from fig(a) the arragement is simply two capacitors in parallel.

So equivalent capacitance

$$C = C_1 + C_2$$

$$= \frac{K_1 \epsilon_0 A/2}{d} + \frac{K_2 \epsilon_0 A/2}{d}$$

$$= \frac{\epsilon_0 A}{d}\left(\frac{K_1 + K_2}{2}\right)$$

(ii) from the fig(b) the arragement is simply two capacitors in series combination.

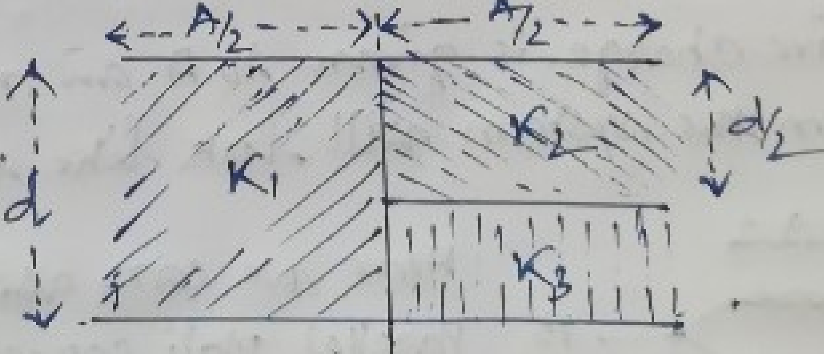
So equivalent capacitance C can be obtained as

$$C = \frac{C_1 C_2}{C_1 + C_2} \qquad \left[\because \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}\right]$$

$$= \frac{\frac{K_1 \epsilon_0 A}{d/2} \cdot \frac{K_2 \epsilon_0 A}{d/2}}{\frac{K_1 \epsilon_0 A}{d/2} + \frac{K_2 \epsilon_0 A}{d/2}}$$

$$= \frac{\epsilon_0 A}{d/2}\left(\frac{K_1 K_2}{K_1 + K_2}\right) = \frac{2\epsilon_0 A}{d} \frac{K_1 K_2}{K_1 + K_2}.$$

* Find the capacitance of the capacitor which is shown in the fig below.



A dielectric with dielectric constant K_1 is filled. The capacitance of this part is

$$C_1 = \frac{K_1 \epsilon_0 A/2}{d} = \frac{K_1 \epsilon_0 A}{2d} \cdots$$

Two dielectrics with dielectric constant K_2 and K_3 are filled in other part. The capacitance of that part is

$$C_2 = \frac{\frac{K_2 \epsilon_0 A/2}{d/2} \cdot \frac{K_3 \epsilon_0 A/2}{d/2}}{\frac{K_2 \epsilon_0 A/2}{d/2} + \frac{K_3 \epsilon_0 A/2}{d/2}}$$

$$= \frac{\epsilon_0 A}{d}\left(\frac{K_2 K_3}{K_2 + K_3}\right)$$

Now C_1 and C_2 are in parallel combination. So equivalent capacitance

$$C = C_1 + C_2$$

$$= \frac{K_1 \epsilon_0 A}{2d} + \frac{\epsilon_0 A}{d}\left(\frac{K_2 K_3}{K_2 + K_3}\right)$$

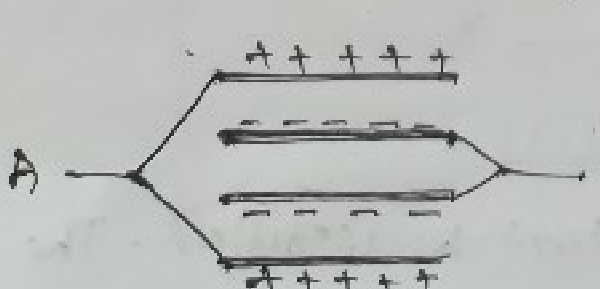
$$= \frac{\epsilon_0 A}{d}\left[\frac{K_1}{2} + \frac{K_2 K_3}{K_2 + K_3}\right]$$

* In the fig below each metal plate has surface area A and separation between two plates is d. Find the capacitance of the system.

(i)

(ii)

(i) If positive charge is given to A and negative charge to the B then the system will look like the fig. below



Here we can assume that two parallel plate capacitor are in parallel combination.

So the capacitance $C = \frac{2\epsilon_0 A}{d}$.

ii) In this case the fig will look like as shown below.

Here we can assume that three parallel plate capacitor are in parallel combination.

So capacitance $C = \frac{3\epsilon_0 A}{d}$.