Economics Honours (Semester II)

Mathematical Economics -I

Revision in Functions

Linear functions: A function y = f(x) of a single variable is linear if it takes the form **$y=f(x) = ax + b$**

where a, b are constants i.e. a & b do not vary as x varies.

Polynomial Functions: A function y = f(x) is called a polynomial function in single variable when expressed in the form

$y = f(x) = a_1x + a_2x^2 + \ldots + a_n x^n$ where a_i, (i=1,..,n) are constants.

Quadratic Functions: A function y = f(x) is called a quadratic function when expressed in the form **$y = f(x) = ax^2 + bx + c$**

where a, b, c are constants.

Quadratic functions in single variables are polynomials raised to the power of 2.

Logarithmic Functions: The natural logarithmic function is denoted by In(x) [log x] and is such that **y = In(x) if and only if $x = e^y$**

[The domain of this function is the set of positive real numbers]

Natural logarithms possess the following properties:

1. In(a x b) = In(a) + In(b)
2. In(a/b) = In(a) - In(b)
3. In(a^n) = nIn(a)
4. In(e) = 1
5. In(1) = 0

Exponential Functions: The general form of an exponential function is

$\mathbf{y = f(x) = a^x}$

where a is some positive constant.

As with logarithmic functions we shall be interested primarily in the exponential function based on Napier's constant e.

When we refer to the exponential function, we shall usually mean the function to take the form $\mathbf{y = e^x}$ **[e = 2.71828]**

Composite Function: A function y = f(x) is a composite function of x if it can be written in the form **y = g(z), where z=h(x), i.e. y =g[h(x)]**

Inverse Function: Two functions of the form **y = f(x) and x =g(y)** are said to be inverse of each other if their composition is equal to the identity function, i.e **g[f(x) =x and f[g(y)] = y**.

Example: If a≠ 0, then y = ax +b and x = [(y-b)/a] are the inverse of each other.

The reciprocal of a function is not an inverse function. Further, not all functions have an inverse.

Implicit function: Suppose there are two variables x and y. A function is said to be an implicit function when it is of the form

z = f(x,y) = k, where k is a constant.

To state clearly an implicit function is generally expressed in both the variables x & y or as a combination of both the variables x & y on the same side of the equation.

Example $\mathbf{x^2 + y^2 = 1}$. The same function when written as $y = \pm\sqrt{(1- x^2)}$ becomes an explicit function.

However, it is not always possible to change an implicit function into an explicit function.

Homogeneous Functions: A homogeneous function f(x) of n variables x_i, (i=1,..,n), is said to be homogeneous of degree k if for all $\alpha > 0$, we have

$$\mathbf{f(\alpha x) = \alpha^k \cdot f(x)}$$

Homothetic Function: A production function is homothetic if it is a monotonic transformation of a homogeneous production.

Example: If **f(x,y) = xy is a homogeneous production function**, then the monotonic transformation of the function say **g(x,y) = xy +1 is said to be a homothetic function**. Here we can write g(x,y) = g[f(x,y), K]. Hence homogeneous implies homothetic but not conversely.

*N.B. Explanations and all theoretical dispositions have been accessed from the following references

References

1. Birchenhall, Chris and Grout, Paul. *Mathematics for Modern Economics*, (1987), Heritage Publishers, New Delhi.