Economics Honours (Semester II)

Mathematical Economics -I

Applications of Partial Derivatives in Economics

Problems in Consumer's Theory

1. Consider the following utility functions

(i) $u(x_1,x_2) = x_1^{1/2} x_2^{1/2}$ (where x_1,x_2 are the two commodities consumed)

(ii) $v(x_1,x_2) = x_1^2 x_2^2$ (where x_1,x_2 are the two commodities consumed)

For each find the marginal utilities and the change in marginal utilities with respect to each commodity. How does the value of marginal utility change for changes in each commodity in both the utility functions mentioned in (i) and (ii).

2. If the demand price function is given by $p=a-bq$, (a, b being positive constants). Find the elasticity of demand as a function of quantity q.

If the demand price function is

$p = ay^c$

where a,c are constants, show that the elasticity of demand is constant.

Problems in Production and Costs

3. Consider the production function $y = f(l,k)$, [y is the output, l is the labour input and k the capital input of a firm]. Given the production function is homogeneous of degree α. Show that the marginal productivities of the function are homogeneous functions of degree $(1-\alpha)$.

4. If the amount of capital k required to produce output q is given by $k = q^2 + q$, find the marginal product of the capital.

5. Consider a production function $y = Al^{\alpha}k^{\beta}$, where y is the output, l is the labour input and k the capital input of a firm.

a. Determine the elasticities of output with respect to labour and capital.

b. Show that the marginal productivity of labour is proportional to the average productivity.

c. Find $MRTS_{lk}$

6.Find the elasticity of substitution 'σ' of the following production functions:

(i) $f(l,k) = Al^{\alpha}k^{\beta}$, where A, α, β are constants

(ii) $f(l,k) = [\alpha l^{z} + (1-\alpha)k^{z}]^{1/z}$, where A, α, z are constants.

7. Find the marginal cost function given the following total cost function:

$TC = a + bq - cq^2 + dq^3$

where a, b, c, d are constants and q is the output.

8. If the firm's average cost of producing q is given by

$AC = \alpha q^n$

where α, n are constants, prove that the marginal cost is (n+1) times the average cost.

Problems in Market Theory

9. Consider a monopolist firm with demand price p(y) for its output y so that its total revenue function is TR = p(y).y

Show that the marginal revenue function MR can be written as

MR = p(1+ 1/e) where e is the elasticity of demand.

*N.B. The problems have been accessed from the following reference

Reference

1. Birchenhall, Chris and Grout, Paul. *Mathematics for Modern Economics,* (1987), Heritage Publishers, New Delhi.