Equations and Identities

I. What is an Equation?

Generally an equation expresses a statement or problem or an expression (usually an algebraic expression) involving one or more variables or unknowns on two sides, namely, the left hand side (LHS) and the right hand side (RHS) of an equality sign (=). Generally solving an equation helps us to arrive at the value of the variable or the unknown.

Example: $5x + 6 = 31$,

Here the algebraic expression on the LHS is '$5x + 6$' and the value given on RHS is'31'. According to the definition, the expression on the LHS and value on the RHS are stated to be equal with the help of the equality sign (=). The value of x can be determined by solving the equation.

II. Types of Equations

- Linear equation in one variable
- Non Linear equation in one variable
- Linear equation in more than one variable
- Non Linear equation in more than one variable

A. Linear equation in one variable

A linear equation in one variable or one unknown is usually of the form $Ax + B = C$, where 'x' is the variable, A, B, C are real numbers and $A \neq 0$. The equation is called linear because the exponent on the variable or the power of the variable is stated to be one. Solving the equation determines the value of the variable 'x'.

Recall the equation in the above example.

It is given,

$5x + 6 = 31$,

Or, $5x = 31 - 6$, [By transposing]

Or, $5x = 25$

Or, $x = 25/5$, [By transposing]

Or, $x = 5$

Thus in our example, the value of $x=5$. It must be pointed out here that in a linear equation of one variable, only a single solution is determined for the variable.

B. Non Linear equation in one variable

An equation is said to be non-linear in one variable when the maximum degree or power of the single variable used in in the equation is more than one. For example, $Ax^3 + Bx^2 + Cx + D = 0$, is a non-linear equation or cubic equation in x where A, B, C, D are real numbers and $A \neq 0$. This is an example of non-linear equation in x, because the maximum degree or power of the variable is more than one. It must be pointed out here the total number of solutions for the variable that is determined depends again upon the maximum degree or power of the single variable. Thus in our example the non-linear equation is a cubic expression in x, we determine three possible solutions of the unknown variable x. These solutions are also called the roots of the equation since they satisfy the equation. Thus a cubic equation in x will have three roots which satisfy the equation.

Quadratic Equation

A special type of non-linear equation in one variable commonly used is the Quadratic Equation. A Quadratic equation is a non-linear equation in one variable of degree '2'. The form of the equation is given by $Ax^2 +Bx + C =0$, where A, B, C are real numbers and $A\neq 0$. The solution of this equation gives two roots of x which satisfy the equation. Sometimes the solution can be determined with the method of factorization. The roots of x can also be determined by the formula;

$$X= \frac{- B \pm \sqrt{(B^2 - 4AC)}}{2A}$$

Example:

Find x, when $5x^2 +6x + 1=0$.

Given $5x^2 +6x + 1=0$, By the method of factorization we have,

Or, $5x^2 +5x +x + 1=0$ [Breaking the middle term]

Or, $5x (x +1) +1 (x+1) =0$, [Taking common]

Or, $(x +1) (5x +1) = 0$. [Taking common]

Hence either $(x +1) =0$, or, $(5x +1) =0$

i.e. either $x = -1$ or, $x = - 1/5$.

Thus the roots of x are -1 and -1/5 of where both satisfy the given equation.

We can obtain the same solutions when we apply the formula to determine the roots of x.

In the equation $5x^2 +6x + 1=0$, A=5, B=6 and C=1

Inserting the values of A, B, C in the formula, the solutions for x is determined as follows,

$x = \frac{-6 \pm \sqrt{(6^2 - 4x5x1)}}{2x5}$,i.e. $x = \frac{-6 \pm \sqrt{16}}{10}$, i.e. $x = \frac{-6 \pm 4}{10}$ i.e. Either x= - 1/5 or x = -1.

C. Linear equation in more than one variable

Suppose there are two variables x and y. The linear equation in two variables takes the form $Ax + By + C = 0$, where A, B, C are real numbers and $A, B \neq 0$. The equation is said to be linear as the power of both the variables x and y is one. The key to solving linear equations in more than one variable is

- there must be as many equations as the number of unknowns or variables present in the equations and
- there is only a single solution for each unknown.

So we can state that

- If there are two unknowns or variables, there must exist a set of two linear equations in the two variables or unknowns and the total number of solutions obtained will be equal to the total number of unknowns.
- In our example if we have to find the solution of x and y, we require two equations in x and y and only a single solution each for the variables x and y would be determined.
- To solve a linear equation in *n* unknowns, we require n equations in *n* unknowns and *n* solutions will be obtained for *n* unknowns
- A set of two equations in x and y used to determine the values of two variables x and y is called simultaneous equations or system of equations.

D. Non-Linear equation in more than one variable

Suppose there are two variables x any. A non-linear equation in the two variables follows the format $Ax^n + By^n + C = 0$, where A, B, C are real numbers; $A, B \neq 0$ and the power $n > 1$.

Examples of non-linear equations in more than one variable:-

Equation of a circle: $(x-h)^2 + (y-k)^2 = r^2$, where (h,k) are the coordinates of the centre of a circle and r is the radius of the circle

Equation of a parabola: $y = ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.

Equation of a hyperbola: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a, b, are real numbers and $a, b \neq 0$.

Equation of a rectangular hyperbola: $xy = C$, where C is a constant.

III. Identities

Identities are equalities that are applicable universally for all values of the variables expressed in the identity.

Examples of identities where a, b, c are variables taking any value

$(a + b)^2 = a^2 +2ab + b^2$

$(a - b)^2 = a^2 -2ab + b^2$

$a^2 - b^2 = (a + b)(a – b)$

$(a + b)^3 = a^3 +3a^2b +3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b +3ab^2 - b^3$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab +2bc +2ca$