Linear Inequatilities

SECTION 1. Meaning of Linear Inequatilities

Generally an equation expresses a statement or problem involving one or more variables on two sides, namely, the left hand side (LHS) and the right hand side (RHS) of an equality sign (=). However it is not always possible to translate a statement or a problem in the form of an equation. We may come across certain problems involving inequality. Inequality is a statement which shows that one quantity is greater (or less) than another quantity. Like '9' is greater than '5' or '2' is less than '6'.

Conventionally, instead of writing in phrases to describe inequality, a sign or symbol is used.

- For the phrase 'is less than', the symbol $<$ is used,
- For the phrase 'is less than or equal to', the symbol $\leq$ is used,
- For the phrase 'is greater than' , the symbol $>$ is used and
- For the phrase 'is greater than or equal to', the symbol $\geq$ is used.

These statements or problems in the linear form involving these signs or symbols are called 'linear inequalitiess' or 'linear inequalities'. For example, statements such as $x < 3$, $x+5 \leq 7$, $2x - 7 > 8$, $3x + 5 \geq 11$ are called linear inequalitiess in one variable. In general, linear inequalitiess in one variable can always be written as $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$ or $ax + b \geq 0$; where a and b are real numbers, $a \neq 0$.

[Adapted from Understanding ICSE Mathematics, Class X by M.L. Aggarwal]

SECTION 2. Weak and Strict Inequalities

We usually distinguish inequatilities between weak and strict inequalities. If we write x y, then we indicate a weak inequality and read the notation as x is no less than y or x is greater than or equal to y. On the other hand if we write $x> y$, then we indicate a strict inequality, namely x is strictly greater than y. Here it must be pointed out that if x is *positive* then $x > 0$, which means x is strictly greater than zero, while if we state that y is non-negative, means $y \geq 0$,which means y can also take the value zero.

[Taken from Mathematics for Modern Economics, by Chris Birchenhall & Paul Grout]

SECTION 3. Rules for Solving Linear Inequatilities

The rules for solving inequatilities in one variable are similar to those for solving an equation in one variable except for multiplying or dividing by a negative number.

One can apply the following rules to linear inequatilities:

1. Add (or subtract) the same number or expression on both sides

Example: If $x < 2$, then $-x > -2$ [Multiplying both sides by -1]

2. Any number (or expression) can be transposed from one side of an inequalities to the other side with the sign of the transposed number (or expression) changed (+ve to –ve and –ve to +ve)

Example: If $2x-1 \geq 5$, then $-4(3x-1) \leq -20$ [Multiplying both sides by -4]

3. Multiply (or divide) both sides by the same positive number. However, when you multiply (or divide) by the same negative number, then the symbol of the inequalities is reversed.

Example: If $-5x \leq 10$, then $x \geq -2$ [Dividing both sides by -5]

[Taken from Understanding ICSE Mathematics, Class X by M.L. Aggarwal]

- **The rules for solving linear inequalities also form the properties of Inequalities.**

Exercises: [Taken from Understanding ICSE Mathematics, Class X by M.L. Aggarwal]

1. Given $x \in \{-3, -4, -5, -6\}$ and $9 \leq 1- 2x$, find the possible values of x.

2. Solve the inequation $3-2x \geq x-12$, given $x \in N$.

3. Solve the following inequations for real x:

(i) $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$

(ii) $\frac{1}{2}\left(\frac{3}{5}x + 4\right) \geq \frac{1}{3}(x-6)$

4. List the elements of the solution set of the inequation $-3< x-2 \leq 9-2x$, $x \in N$.

5. Find three largest consecutive natural numbers such that the sum of one-third of first, one-fourth of second and one-fifth of the third is almost 25.

References:

1. Aggarwal, M. L. (2005). *Understanding ICSE Mathematics, Class X* (2019 ed.), Avichal Publishing Company, New Delhi.
2. Birchenhall, Chris and Grout, Paul. *Mathematics for Modern Economics*, (1987), Heritage Publishers, New Delhi.