Review of Definitions

Before continuing, it is helpful to review some definitions:

- A **walk** is a sequence of alternating vertices and edges. A **trail** is a walk with no repeated edges and a **path** is a walk with no repeated vertices.
- A **circuit** is a closed trail; a trail that starts and stops at the same vertex.
- An **Eulerian circuit** in a graph is a circuit that contains every edge of the graph.
- An **Eulerian trail** in a graph is a trail that contains every edge of the graph.
- A graph is called an **Eulerian Graph** if it has an Eulerian circuit.

Euler's Theorem

Theorem (Euler's Theorem)

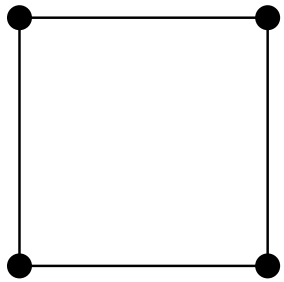
Let G be an connected, undirected graph. G has an Eulerian circuit if and only if every vertex in G has even degree. G has an Eulerian trail if and only if G has exactly two vertices with odd degree.

Basic Algorithm to find an Eulerian Circuit in G:

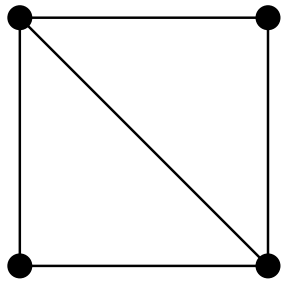
1. Let C be an empty circuit in G, assign $G_0 = G$, and set $k = 0$.
2. Find any circuit C_k in G_k.
3. Merge circuit C_k into circuit C.
4. Construct graph G_{k+1} from G_k by removing edges in circuit C_k.
5. If C does not contain all edges in G then increment k and go to Step 2.

Finding Eulerian Circuits and Trails

Which of the following graphs have an Eulerian circuit? If you can't find a Eulerian circuit, can you find an Eulerian trail?



A



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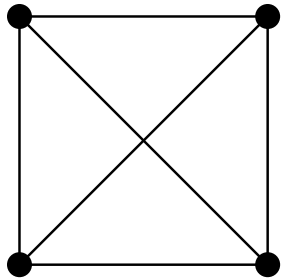

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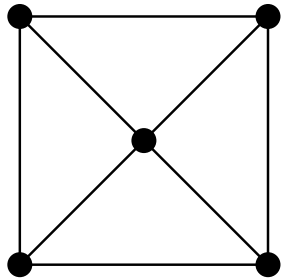
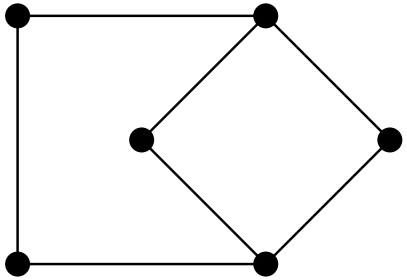


D

Finding Eulerian Circuits and Trails

How about these? Any Eulerian circuits or trails?



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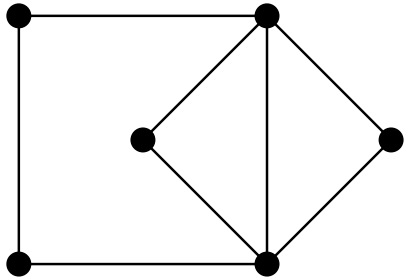

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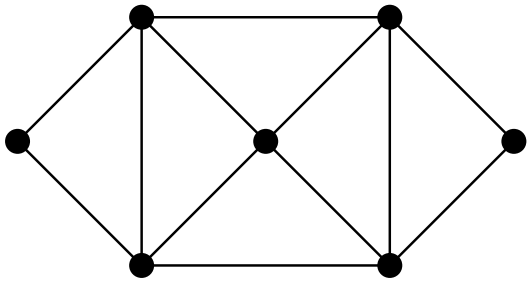

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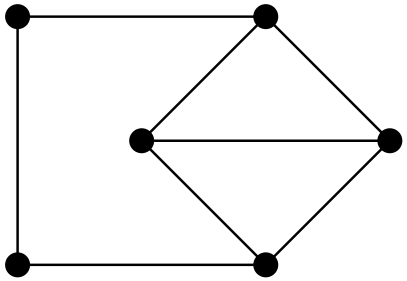

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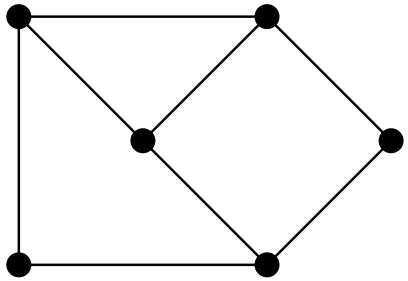
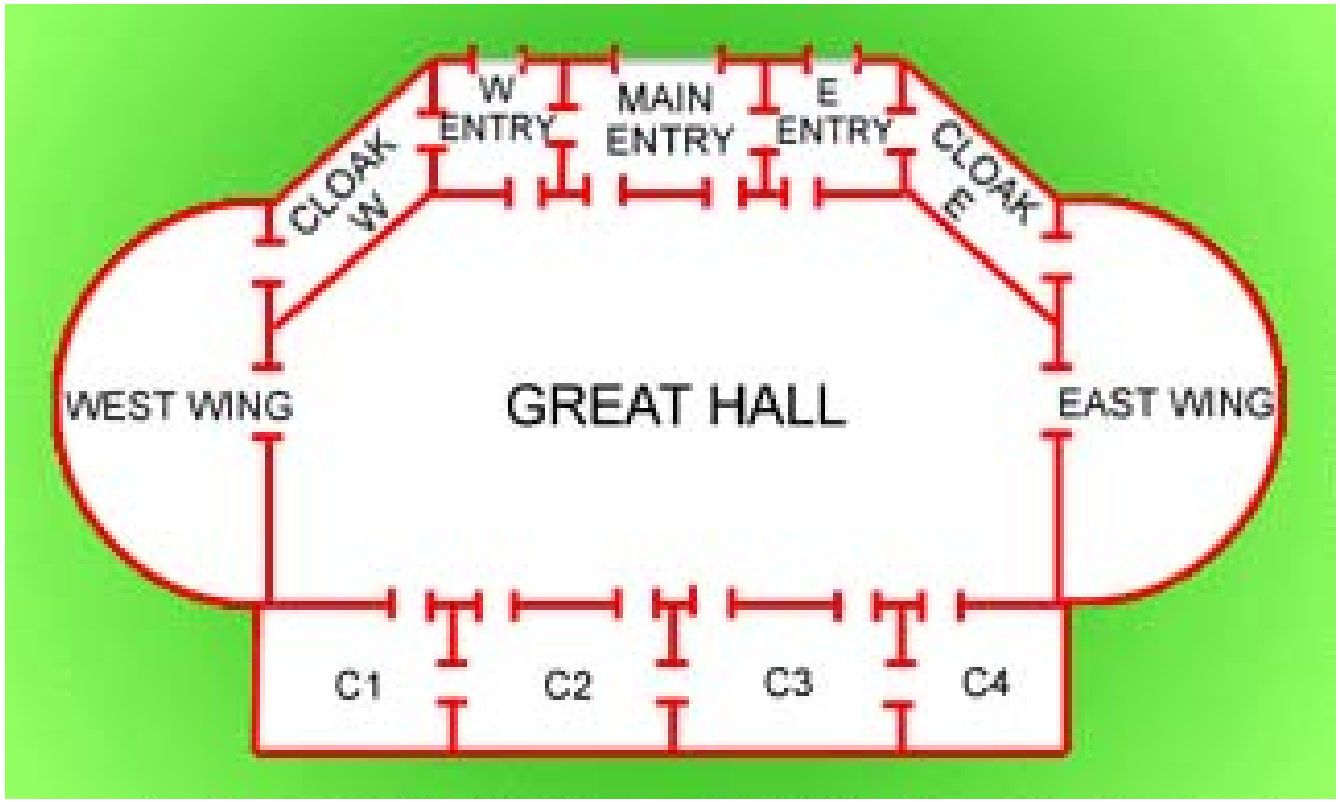


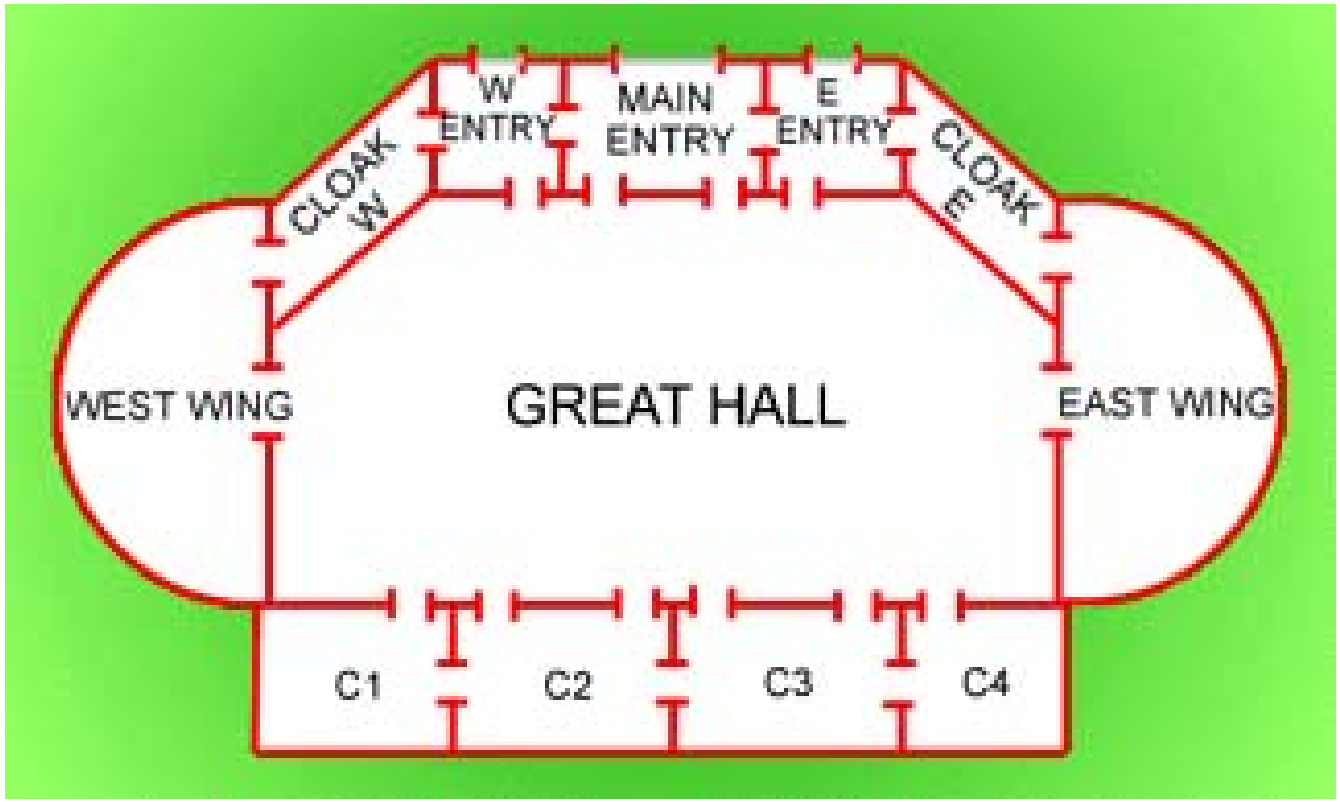
H

Finding Eulerian Circuits and Trails

Consider the floor plan shown here. Is it possible to walk through and around this building passing through each and every doorway exactly once?

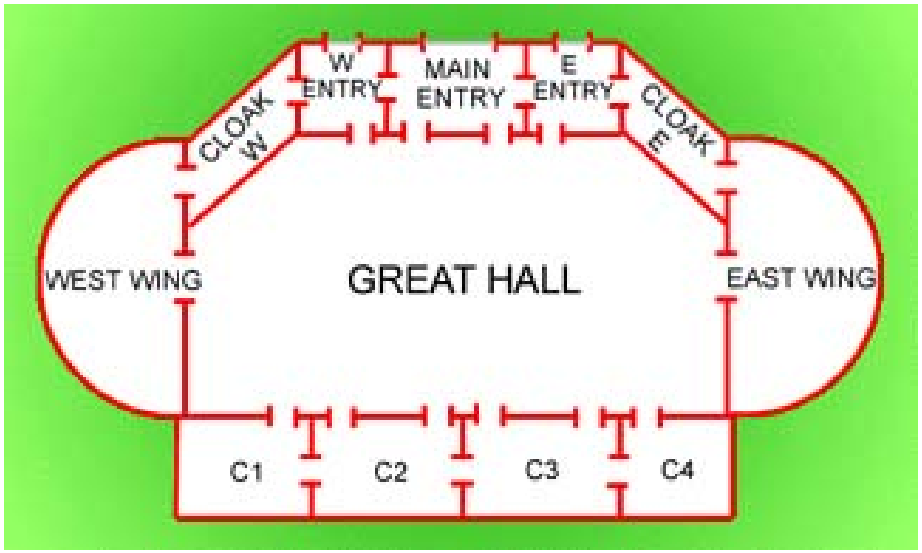


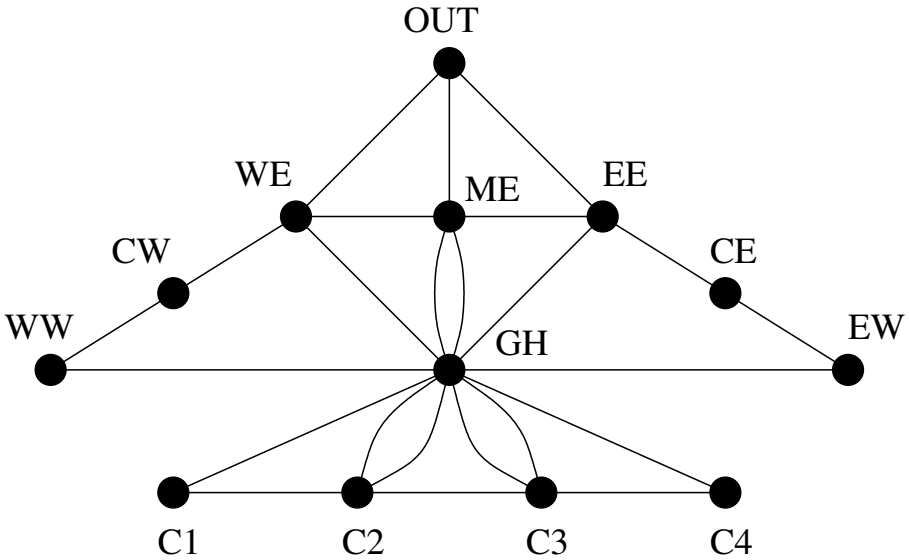
Finding Eulerian Circuits and Trails



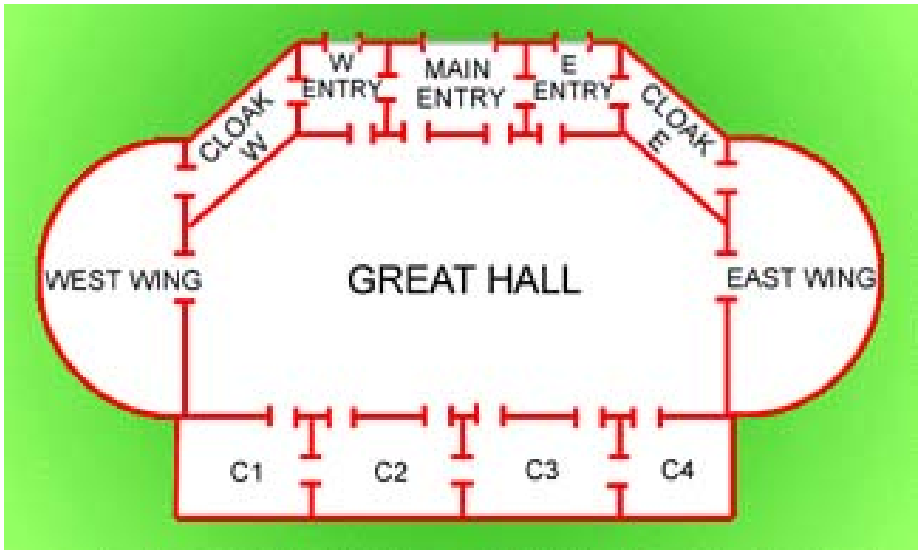
1. Start by creating any circuit.
2. Pick any vertex with unused edges and find a circuit using it. Merge this new circuit into the previously found circuit.
3. Continue the above steps until an Eulerian circuit or trail is found.

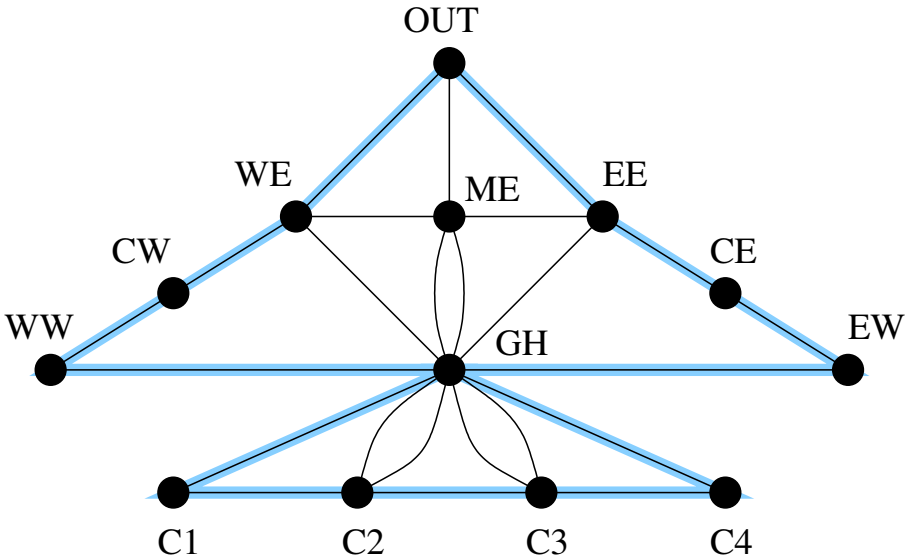
Finding Eulerian Circuits and Trails



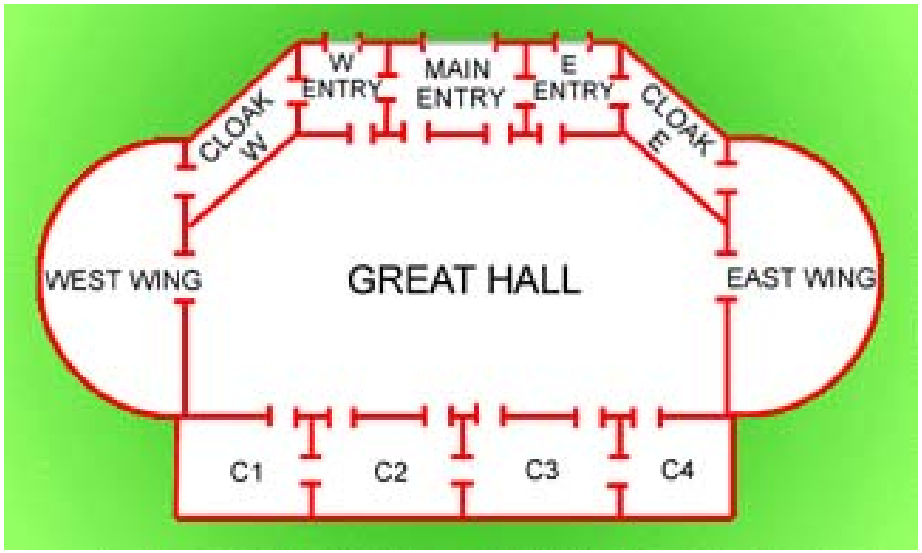


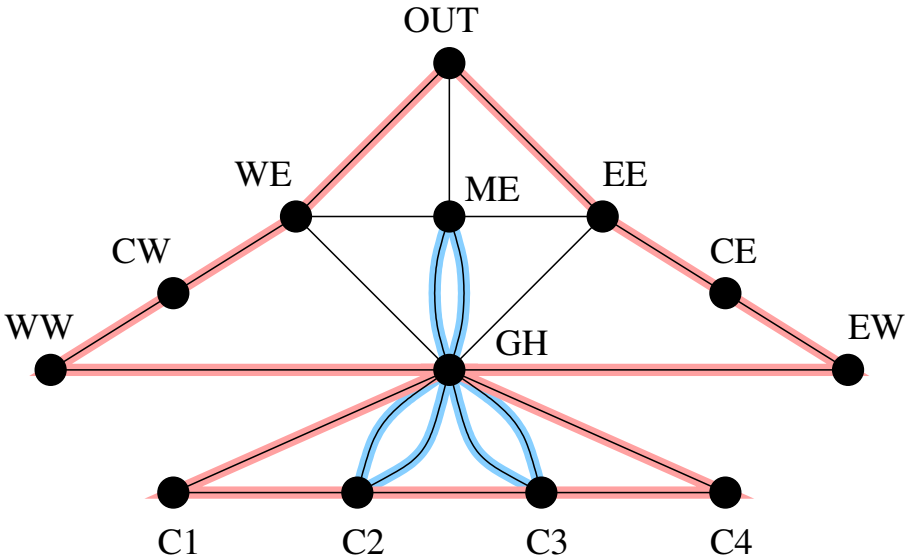
Finding Eulerian Circuits and Trails



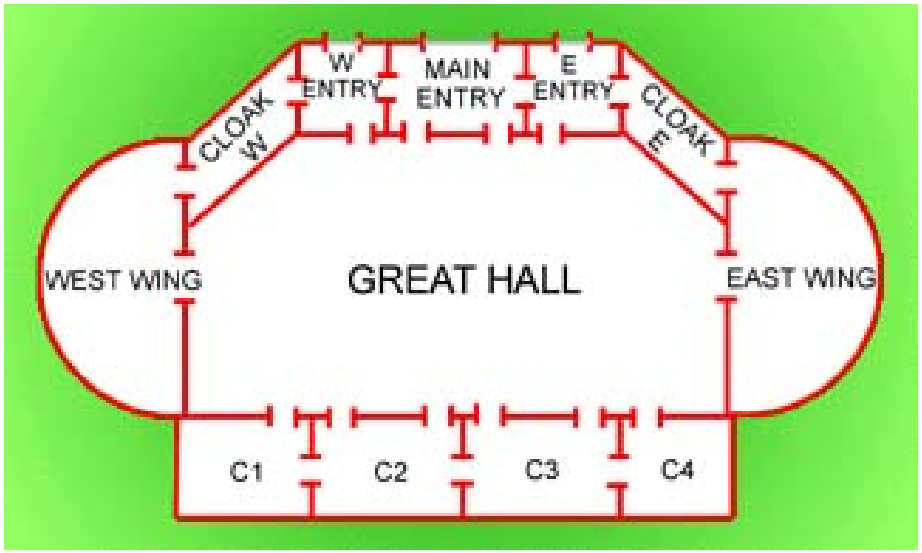


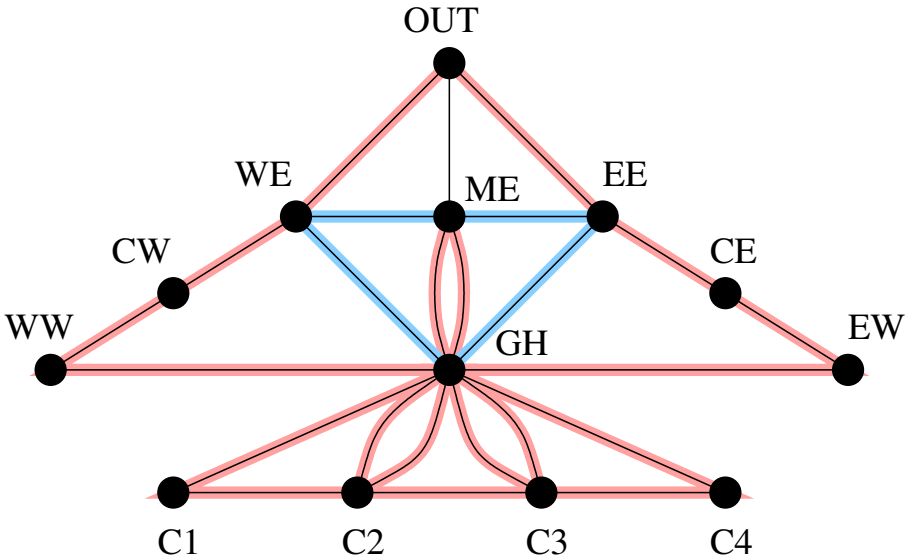
Finding Eulerian Circuits and Trails



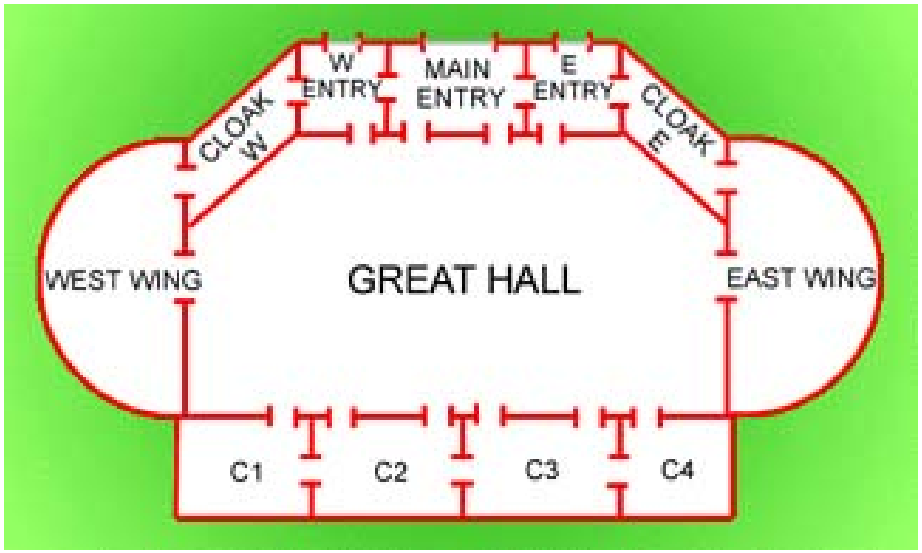


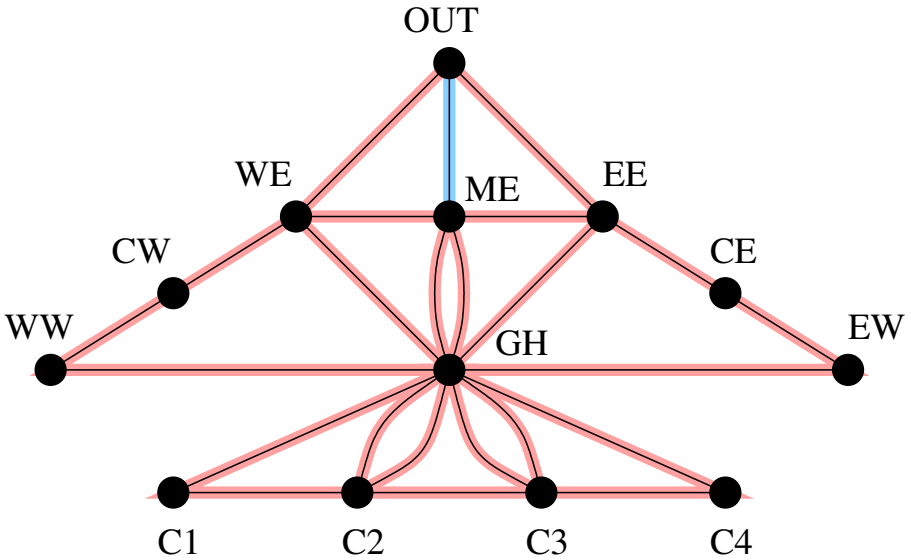
Finding Eulerian Circuits and Trails





Finding Eulerian Circuits and Trails





Both OUT and ME have an odd number of incident edges; we can start at one and end at the other. Thus, we have found an Eulerian trail.