Raniganj Girls' College
Fifth Semester Mathematics (Hons)
Code: DSE1
Linear Algebra

1. (a) Let $T: R^3 \quad R^3$ be a linear operator defined by $T(a, b, c) = (3a , a – b , 2a +b+ c)$.
Prove that $(T^2 – I)(T – 3I) = 0$.

(b) Let $T: R^2 \quad R^2$ be a linear transformation such that $T(2,3) = (1,0)$ and $T(3,2)= (1,-1)$.
Find the matrix representation of T.

(c) Let $T: R^3 R^3$ be a linear transformation defined by $T(x, y, z) = (x+2y - z , y + z, x + y-2z)$.
Find the basis and dimension of (i) image of T, (ii) kernel of T.

2. (a) If $T: V \quad W$ be a linear transformation, prove that R(T) is a subspace of W.

(b) Let $T: V W$ be a linear transformation defined by $T(x, y, z) = (x + y , x - y, 2x + z)$. Find the rank and nullity of T.

(c) Find the matrix of the linear transformation $T:V_2(R) V_3(R)$ defined by $T(x, y) = (x + y , x, 3x - y)$ with respect to bases $B_1 = \{(1,1), (3,1)\}$ and $B_2 = \{ (1,1,1) ,(1,1,1) ,(1,0,0) \}$.

3. (a) Define an inner product space. For any vectors α , β in an inner product space V ,

prove that $\| \alpha +\beta \| \leq \|\alpha \| + \| \beta \|$.

(b) Prove that an orthogonal set of non zero vectors is linearly independent.

(c) Apply the Gram-Schmidt orthogonalization process to find an orthonormal basis for the subspace of R^4 spanned by the vectors $v_1 = (1,1,1,1)$, $v_2 = (1,2,4,5)$, $v_3 = (1,-3,-4,-2)$.

8 (a) If V is an inner product space , then for any vectors α , β in V and any scalar C , prove that

(i) $\|C\alpha\| = |C|\,\|\alpha\|$ (ii) $|(\alpha, \beta)| \leq \|\alpha\|\,\|\beta\|$

(b) Prove that every finite dimensional inner product space has an orthonormal basis**.**

(c) Find the QR-decomposition of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

9. (a) Diagonalize the matrix A, given that $A = \begin{pmatrix} -19 & 6 \\ -47 & 16 \end{pmatrix}$

(b) Find the minimum and maximum values of $Q(x) = 9x^2 + 4y^2 + 3z^2$ subject to the constraint $x^T x = 1$.

(c) Find the singular value decomposition of $A = \begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}$

10.(a) Make a change of variable x = p y that transforms the quadratic form $8x_1^2 - x_1^2 x_2^2 - 5x_2^2$ into a quadratic form with no cross product term .

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(b) Orthogonally diagonalize the matrix $A = \begin{pmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{pmatrix}$

(c) Find the singular value decomposition of $A = \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}$

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