Raniganj Girls College

Department of Mathematics

UG Examination

5^{th} Semester

Discipline: Mathematics

Course Name: Elements of Topology and Functional Analysis

Course Code: BSCHMTMDSE501

Full Marks: 40 **Time : 2 hrs**

Q.No		Marks
1.	Answer any five questions:	5×1
(a)	Find the closure of the set S={ $1+\frac{1}{n}$: n∈ N } in usual topology on R.	
(b)	Let X={a}. Then, what are the difference between discrete topology and indiscrete topology on X?	
(c)	Define limit point compactness in a topological space.	
(d)	Define Metric Topology.	
(e)	What is separated sets in a topological space?	
(f)	Which is finer topology between R_l- lower limit topology in Real line and R- standard topology in Real line?	
(g)	Define Banach Space.	
(h)	State Parallelogram law in Hilbert Space.	
2.	Answer any five questions:	5×2
(a)	Define Neighbourhood Basis of a topological space (X,ρ).	
(b)	Define First Countable Space. Give an example.	
(c)	Define Basis for a Topology.	
(d)	Whether, R-the real line is connected ?	

Q.No		Marks
(e)	Define Lindelof Space. Give an example.	
(f)	Define Pre-Hilbert Space.	
(g)	What is Orthonormal sets in a Hilbert Space?	
(h)	Check, whether l_p , for $p\neq 2$ is an inner product space or not ?	
3.	Answer any three questions:	3×5
(a)	Prove that, in a topological space (X,ρ) , (i) Any set C, containing a dense set D, is a dense set. (ii) If A is a dense set, and B is a dense on A, then B is also a dense set.	2.5+2.5
(b) (i)	Define T_0, T_1 and T_2 space in a topological space (X,ρ).	3×1
(ii)	Let, $X=\{a,b,c\}$, and $\rho=\{\phi, \{a\},\{a.b\}, X\}$. Then, prove that (X, ρ) is a T_0 space, but not a T_1 space.	2
(c)	Let, A be a connected subset in a topological space (X, ρ). If $A \subset B \subset \bar{A}$, then prove that B is also Connected.	
(d)	Prove that, any closed subspace of a compact space is compact.	
(e)	Let, $e_1, e_2, \ldots, e_n, \ldots$ be an orthonormal sequence in a Hilbert space H. Then Prove that, for every $x \in H$, $\sum_{i=1}^{\infty} \mid (x, e_i) \mid^2 \leq \|x\|^2.$	
4.	Answer any one questions:	1×10
(a) (i)	If f is a homeomorphism of (X,ρ_1) onto (Y,ρ_2) and g is a homeomorphism of (Y,ρ_2) onto (Z,ρ_3), then prove that, $g \circ f$ is a homeomorphism of (X,ρ_1) onto (Z,ρ_3)	5
(ii)	Prove that, every Metrizable space is Normal.	5
(b) (i)	Prove that, the image of a connected space under a continuous map is connected.	5
(ii)	Prove that, compactness implies limit point compactness in a topological space, but the converse is not true.	5

(c) (i)	Prove that, a Banach space is a Hilbert space if and only if the parallelogram law holds.	6
(ii)	Prove that, the space **C[a,b]** is not a Hilbert space.	4