Raniganj Girls' College

Department of Mathematics

5th semester

Topology

Some questions from Topology. Each question carry 5 marks

1. **Define filter and give one example of it.If X is an infinite set and F is the collection of all non empty subsets of X whose complements are finite then prove that F is a filter on X.**
2. **Prove that the intersection of two filters is a filter. Verify whether union of two filters is a filter or not.**
3. **Let F be a filter on a non empty set X and A be a subset of X. Then prove that there exists a filter F' finer than F such that A ε F' if and only if A$\cap$ $G \neq \emptyset$ *for every* $G \varepsilon F$.**
4. **Let F_1 and F_2 be any two filters on X and B_1 and B_2 be their bases respectively.Prove that F_2 is finer than F_1 if and only if every member of B_1 contains a member of B_2.**
5. **Define Directed set and net. Give examples.**
6. **A topological space (X,τ) is Hausdorff if and only if every convergent filter on X has a unique limit.**
7. **If d is a metric for a non empty set X, show that function $d^*(x,y) = d(x,y)/1+d(x,y)$ is also a metric for X.**
8. **Show that C[a,b],the collection of all continuous real valued functions defined on [a,b] is a complete metric space.**
9. **Prove that a complete metric space is a set of second category.**
10. **Prove that If a metric space is complete then every nested sequence of non empty closed subsets {F_i } ith diameter tending to zero be such that F= $\cap$ F_i contains exactly one point.**
11. **Define the notions "Uniformly boundedness and equicontinuity" .**
12. **If a set M, subset of C[a,b] is compact in C[a,b] then show that the aggregate of functions x(t) in M are uniformly and equi continuous.**
13. **Let X be a metric space and A is a subset of X then prove tha if A is compact, A is totally bounded.**
14. **Give one example to justify that every bounded set is not totally bounded.**
15. **Prove that if X is a complete and A is totally bounded the A is compact in X.**
16. **Let X be a complete metric space and let T be an operator mapping X into itself such that**

$d(Tx,Ty) \leq \alpha\, d(x,y)$,$0\leq \alpha < 1$. Then T has a unique fixed point in X.