Raniganj Girls' College Department of Mathematics Subject-Mathematics(Honours) Paper- CC-12

Some Important questions

1. each questions of 2 marks

(a) Write down Newton's laws of motion and indicate which one asserts the essence of an inertial frame.

(b) Dose the velocity of light remain invariant under Galilean transformations? Justify your answer.

(c) Distinguish between body forces and surface forces. Give example of each.

(d) How do you define a body to be a 'continuous medium'?

(e) What is the direction of the stress vector on a surface element in a viscous fluid at rest? Justify your answer.

(f) What is virtual work?

(g) If an 'inertia matrix' were such that all the off diagonal elements are zero, what can you say about the diagonal elements and the co-ordinate axes involved?

2. each questions of 5 marks

(a) For a system of n particles, show that the rate of change of angular momentum of the system about the fixed point o is equal to the sum of the moment about o of the external forces. Clearly state that the nature of the inertial forces in the system for which the above result holds.

(b) Show that the angular momentum of a system of particles about a fixed point o is equal to the sum of angular momentum of the total mass placed at the centre of mass about o and the angular momentum of the system about its centre of mass.

(c) The straight line 4x + 3y = 5 meets the rectangular axes *OX*, *OY* at *A* and *B* respectively. If the forces

X, Y, Z act along the lines *OB*, *OA* and *AB* find the magnitude of the resultant and the equation of the line of action.

(d) A uniform road *OA* length *a* ,free to turn about its end *O* ,revolves with uniform velocity ω ,about the down ward vertical *OZ*. Using D'Alembert's principle, or otherwise, show that the inclination is either

zero or
$$\cos^{-1}(\frac{3g}{2a\omega^2})$$
.

Also find the reaction at the hinge.

c. (i)Show that any system of coplanar forces, acting on a rigid body can be reduced to a single resultant force acting at an arbitrary point O and a single resultant couple whose moment is equal to the algebraic sum of moment of the given forces about O.

(ii) If p be the pressure, ρ the density and \vec{F} the external force per unit mass at a point (x, y, z) of a fluid in equilibrium, then show that $dp = \rho(Xdx + Ydy + Zdz)$.

Where X, Y, Z are the rectangular Cartesian components of \vec{F} .

Hence, show that the lines of force intersect the equi-pressure surfaces orthogonally. 6+(4+2)

 $3 \times 6 = 18$

3. Answer any three questions:

a. Show that any system of coplanar forces, acting on a rigid body can be reduced to a single resultant force acting at an arbitrary point O and a single resultant couple whose moment is equal to the algebraic sum of moment of the given forces about O.

b. If a given volume of fluid is under forces whose components per unit mass are $\lambda y(a-z)$, $\lambda x(a-z)$, μxy ,

show that the density must be proportional to $\frac{1}{xy(a-z)}$. Determine its pressure.

c. A force parallel to the z-axis acts at the point (a, 0, 0) and an equal force perpendicular to the z-axis acts at the point (-a, 0, 0). Show that the central axis of the system lies on the surface

$$z^{2}(x^{2} + y^{2}) = (x^{2} + y^{2} - ax)^{2}.$$

d. A heavy homogeneous liquid is rotating uniformly about a vertical axis under gravitational force. Show that the free surface is a paraboloid of revolution.

e. A fine circular tube of radius *a* contains liquid, filling half of its circumference and turns about the vertical diameter with uniform angular velocity. Prove that the liquid will just separate, if the angular

velocity is
$$\sqrt{\frac{2g}{a}}$$
.