Raniganj Girls' College

Department of Mathematics

UG 4th Semester Examination

Award: BSC(Hons)

Discipline: MATHEMATICS

Course Type: GEC

Course Code: BSCHMTMGE401

Course Name: Geometry and Vector Analysis

Full Marks: 50 **Time: 2 hour**

Q.No		Marks
1.	Answer any five questions:	1×5
(a)	Determine the nature of the conic $\frac{l}{r}$ = 4 - 5cos θ.	
(b)	Find a point on the conic $\frac{l}{r}$ = 1 – e cos θ having the least radius vector.	
(c)	What type of curve represented by $2x^2 + 4xy + y^2 = 0$?	
(d)	What does the equation $x^2 + y^2 = 4$ represent in 3D geometry?	
(e)	What does the equation $x^2 = 4y$ represent in 3D geometry?	
(f)	What type of cone represented by the equation ayz + bzx + cxy = 0?	
(g)	Find the equation of the surface obtained by revolving the line $y - mx = 0,\ z = 0$ about x axis.	
(h)	Write down the equation of a circular cylinder and of a circle on it.	
(i)	If $\vec{a} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\vec{b} = 4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$, then find $\lvert 3\,\vec{a} + 2\,\vec{b} \rvert$	
(j)	Find the acute angles that the line joining the points (1, -3, 2) and (3, -5,1) makes with the coordinate axes.	

Q.No		Marks
2.	Answer any five questions:	2×5
(a)	If $ax + by$ and $cx + dy$ are changed to $AX + BY$ and $CX + DY$ respectively for rotation of axes, show that $ad - bc = AD - BC$.	
(b)	Show that the equation $x^2 + 6xy + 9y^2 + 10x + 30y + 25 = 0$ represents a pair of coincident straight lines.	
(c)	Show that the triangle formed by the lines $x + y = 0\,; 3x + y = 4\,; x + 3y = 4$ is isosceles and obtuse angled.	
(d)	Do the lines $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ and $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ intersect?	
(e)	Find the image of the point (-3,8,4) in the plane $6x - 3y - 2z + 1 = 0$.	
(f)	Find the equation of the straight line passing through the point (1,-1,3) and perpendicular to the straight line $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1}$.	
(g)	Find the angles (in degrees) that the line $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{0}$ makes with the axes.	
(h)	If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three unit vectors satisfying $\vec{a} + \vec{b} + \vec{c} = \theta$, then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$	
(i)	Show by the vector method that, the diagonals of a rhombus are at right angles.	
(j)	If $\vec{a}$ and $\vec{b}$ are non-zero vectors such that $\vec{r}.\vec{a} = 0$ and $\vec{r} \times \vec{a} = \vec{b}$, then find the value of $\vec{r}$.	
3.	Answer any three questions:	3×5
(a)	If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines equidistant from origin, show that $f^4 - g^4 = c(bf^2 - ag^2)$.	
(b)	Find the pole of the focal chord of the parabola $y^2 = 4x$, passing through the point (9,6) with respect to the parabola $y^2 = 8x$.	

Q.No		Marks
(c)	Find the equation of the plane through the point (2,-1,0), (3,-4,5) and parallel to the line $2x = 3y = 4z.$	
(d)	Show that the plane $8x - 6y - z = 5$ touches the paraboloid $3x^2 - 2y^2 = 6z$. Find the point of contact.	
(e)	Prove that the lines $\vec{r} = \vec{a} + t(\vec{b} \times \vec{c})$ and $\vec{r} = \vec{b} + s(\vec{c} \times \vec{a})$ are coplanar if $\vec{a}.\vec{c} = \vec{b}.\vec{c}.$	
(f)	Show that $\vec{F} = (2x - yz)\hat{\imath} + (2y - zx)\hat{\jmath} + (2z - xy)\hat{k}$ is irrotational.	
4.	Answer any one questions:	1×10
(a)		
(i)	Find the point of inter-section of the two tangents at α and β to the conic $\frac{l}{r} = 1 + \cos\theta$.	5
(ii)	For what value of λ does the equation $\lambda xy - 9x + 8y - 12 = 0$ represent a pair of straight lines?	5
(b) (i)	A Sphere of constant radius k passes through the origin and meets the axes A, B, C. Prove that the centeroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$.	5
(ii)	Find the shortest distance between the straight lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-5}{4} = \frac{y-4}{4} = \frac{z-5}{5}$. Hence show that the two straight lines are coplanar.	5
(c) (i)	Find the velocity and acceleration, when a particle moves along the curve $x = t^3, y = t^2, z = t$ at $t = 1$ along the direction of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$.	5
(ii)	Prove that, $[\vec{\alpha} + \vec{\beta} \;\; \vec{\beta} + \vec{\gamma} \;\; \vec{\gamma} + \vec{\alpha}] = 2\,[\vec{\alpha} \;\; \vec{\beta} \;\; \vec{\gamma}]$.	5