Raniganj Girls' College

Department of Mathematics

3rd semester(Honours)

Subject-Mathematics

Pape-CC5

(Introduction to Metric Space)

Some Important Question

1. questions of 2 marks each

(a) If (X, d) is a connected metric space, prove that either X is a single-pointic set or it is an infinite set.

(b) If d is a metric on X, then show that $\frac{d}{1+d}$ is also a metric on X.

(c) If $\{x_n\}$ and $\{y_n\}$ are Cachy sequences in a metric space (X, d) then show that $\{d(x_n, y_n)\}$ is a convergent sequence.

(d) Let $f : c[a,b] \to R$ (the space of reals with usual metric) be defined by $f(x) = x(t_0)$, where t_0 is a fixed real number. Prove that f is continuous.

(e) Give an example to show that intersection if an infinite number of non-empty open sets is not an open set.

2. questions of 5 marks each

a. Let (X, d) be a metric space, and let

$d^*(x, y) = \frac{d(x, y)}{1+d(x, y)}$, for all x, y in X.

Then prove that d^* is a bounded metric on X which is equivalent to d.

(b) Defined Fixed-point mapping. Let T be a mapping from a metric space (X, d) to itself. Prove that If T is contraction on X, then T is continuous on X

(c) Prove that in a metric space (X, d), a subset $F \subset M$ is closed if and only if its complement is open.