Raniganj Girls' College
Department of Mathematics

3rd Semester Mathematics(Hons)

Paper : Numerical Analysis

Code: C-7

1. (a) Explain geometrically the method of false position to approximate a zero of a function. Construct an algorithm to implement the method of false position.

   (b) Define order of convergence of an iterative method for finding an approximation to the location of a root of f(x) = 0. Find order of convergence of Newton's method.

   (c) Perform three iterations of secant method to approximate a root of the equation

   $X^3 - 2x - 5 = 0$, considering $p_0 = 1$ and $p_1 = 2$

2. (a) Perform three iterations starting from $p_0 = 1$ of the fixed point iteration scheme for $g(x) = e^{-x}$ Construct an algorithm to implement the fixed point iteration scheme.

(b) Verify that the function $f(x) = \sin x$ has a zero in the interval (3, 4). Perform three iterations of bisection method and verify that each approximation satisfies the theoretical error hound. The exact location of the zero is $p = \pi$

(c) Perform four iterations of Newton's method to find approximate value of

$\sqrt[7]{3}$

using equation $x^7 = 3$ and starting approximation as 1.

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $Ax = [-3 \quad -12 \quad 6]^T$.

(b) Perform three iterations of Jacobi method to solve the system of equations, for the given coefficient matrix and right hand side vector, starting with the initial vector $x^{(0)} = 0$,

$$\begin{pmatrix} 3 & -1 & 1 \\ 2 & -6 & 3 \\ -9 & 7 & -20 \end{pmatrix}, \begin{pmatrix} 4 \\ -13 \\ 7 \end{pmatrix}$$

(c) Starling with initial vector $x^{(0)} = 0$, perform three iterations of Gauss Seidel method to solve the following system of equations :

$2x - y = -1, -x + 4y + 2z = 3, 2y + 6z = 5.$

4. (a) Define the forward difference operator ($\triangle$) and average operator (μ). Prove that Newton Divided difference $f[x_0, x_1, x_2, \ldots x_n]$

$$= \frac{1}{n!\, h^n} \triangle^n f_0$$

(b) The following data set was taken from a polynomial of degree at most five. Find the polynomial.

x	—2	—1	0	1	2	3
f(x)	39’	3	—1	—3	—9	—1

(c) Define the backward difference operator (∇) and central difference operator (∂). Prove.that

(i) $\mu\delta \doteq \frac{1}{2}(\Delta + \nabla)$

(ii) $\nabla = \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$

5. (a) Apply Euler’s method to approximate the solution of the initial value problem.

$$\frac{dx}{dt} = 1 + \frac{x}{t}, \quad 1 \le t \le 6, \quad x(1) = 1$$

(b) Evaluate $\int_0^1 \tan^{-1} x \, dx$ using

(i) Simpson’s one third Rule (ii) Trapezoidal Rule

(c) Find the forward difference polynomial that fits the data.

x	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.00	2.28

Hence interpolate at x = 0.25.

6. (a) Define degree of precision of a quadrature rule. State Trapezoidal’s rule for the evaluation of $\int_a^b f(x)\,dx$ and verify that it has degree of precision 1.

P.T.0.

(b) Verify that the forward difference approximation :

$$f'(x_0) = \frac{1}{2h}\left(-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h)\right)$$

For the first order derivative provide the exact value of the derivative, regardless of the value of h. for the function f(x) — l, $f(x) = x$, $f(x) = x^2$ but not for the function $f(x) = x^3$.

(c) If $f(x) = \frac{1}{x^2}$ then evaluate Newton Divided difference $f[a, b, c, d]$.