**Raniganj Girls' College**

**Department of Mathematics**

**SEM-ii, Mathematics(hons)**

**Course code: BSCHMTMC201**

**Course name: Real Analysis**

**F.M.-40** **Time:- 2 hours**

1. Answer any five question: 5 X 1=5
   a. State Bolzano Weierstrass theorem of sets.
   b. Using order properties of R, prove that if a>0, b>0, => a + b >0 a,b$\in$ R
   c. Using density property of R, prove that there exist an irrational number between two real numbers x,y where x<y.
   d. Find the derived set of bounded open interval (a,b)
   e. Is set of all integers called countably infinite set? Why?
   f. When a set is said to be compact?
   g. Is the series $\sum_{n=1}^{\infty} \frac{\mathrm{n}}{\mathrm{n}+1}$ is convergent?
   h. State Leibnitz's test of convergence for an alternating series.

2. Answer any five question: 5 X 2=10
   a. Show that an open interval (a,b) is an open set.
   b. If S is a closed subset of a compact set K in R, then prove that S is compact.
   c. Let S={ $\frac{1}{m}+\frac{1}{n}$ ; $m,n \in N$ }, show that 0 is a limit point of S.
   d. Prove that $\{X_n\}$ , where $X_n = \frac{1}{n+1}+\frac{1}{n+2}+\cdots\frac{1}{2n}$ is a monotone increasing sequence.
   e. Show using leibnitz's test the series 1- $\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ is convergent.
   f. Find the upper and lower unit of the sequence $\{X_n\}$ where $X_n$= $(-1)^n\left(1+\frac{1}{n}\right)$, n ≥ 1
   g. Is arbitrary intersection of open sets always an open set? If not give an example.

h. Show that the series $\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}$ ... diverge when 0 < P < 1.

3. Answer any three question : 3*5=15

a. (I) Prove that every absolutely convergent series is convergent.

(II) A series is convergent implies it is absolutely convergent ---- is it true? Give an example for your support. 3+2

b. (I) What is Cauchy's condensation test ?

(II) Use it to test the convergency of the series $\sum_{1}^{\infty} \frac{1}{n^p}$ , p > 0 2+3

c. (I) Prove the ratio test; A positive series $\sum U_n$ converges if

$\lim_{n\to\infty}\left(\frac{U_n+1}{U_n}\right)$ < 1 and diverges if $\lim_{n\to\infty}\left(\frac{U_n+1}{U_n}\right)$ > 1 2+3

(II) Applying this, examine the given series for convergence $\frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^3}+$ ...

d. (I)Let S be a non empty bounded subset of R with sup S=M and inf S=m ,prove that the set T= $\{|x-y|: x, y \in S\}$ is bounded above and supT=M-m.

(II) State the Archimedean property of R. 4+1

e. (I) Show that the set of rational number is countable.

(II) Let S={1,1/2,1/3,... }. Find the set int s,if S is open set? 4+1

4. Answer any one question : 1*10=10

a. (I) what is integral test?

(II) P.T the series $\sum_{n=2}^{\infty} \frac{1}{\mathrm{n(logn)^{\wedge}P}}$ converges when p>1 and diverges p≤ 1.

(III) Using integral test, to show the convergency of the series 2+5+3

$\frac{1}{2}+\frac{2}{5}+\frac{3}{10}+. \quad . \quad . \quad .$

b. (I) prove that a monotone decreasing sequence ,it bounded below is convergent and it converges to the g.l.b .

(II) Applying the above theorem test the convergency of the sequence $\left\{\frac{1.3.5.7.......(2n-1)}{2.4.6.8.........2n}\right\}$

(III) Prove that if $\{X_n\}$ is a sequence of non zero terms and if $\lim_{n\to\infty} S_n=\infty$ then $\lim_{n\to\infty} 1/S_n = 0$ . 4+3+3