Raniganj Girls' College

Department of Mathematics

UG Examination

1st **Semester**

Discipline: Mathematics

Course Name: Differential Calculus.

Course Code: BSCHMTMGE101

Full Marks: 40 **Time : 2 hrs**

Q.No		Marks
1.	Answer any five questions:	5×1
(a)	Find $\lim_{x\to 0}\frac{1}{x}.\log(1+x)$.	
(b)	If $f(x) = 1 - x, x \in R$. Check whether $f(x)$ is monotonic increasing or monotonic decreasing?	
(c)	State Darboux's Theorem.	
(d)	State Rolle's Theorem.	
(e)	Find f_x , f_y if $f(x,y) = \sin^{-1}\frac{y}{x}$.	
(f)	Find the pedal equation of $r = e^{\theta}$.	
(g)	Define Continuity of $f(x,y)$ at a point (a,b).	
(h)	$f(x) = \lvert x\rvert$, is the function differentiable at $x = 0$?	
2.	Answer any five questions:	5×2
(a)	Evaluate $\lim_{x\to 0+} x.\log(x)$.	
(b)	Show that the function $f(x) = \sin\frac{1}{x}$; $x \neq 0$ $= 0$; $x = 0$ Is not continuous at $x = 0$.	

Q.No		Marks
(c)	Find y_n , where $y = cos^3 x$.	
(d)	Find $\frac{ds}{dr}$, for the curve $r = a\theta$.	
(e)	What is Singular Point on a curve?	
(f)	If $u = \sqrt{xy}$, find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.	
(g)	Find the radius of curvature of $y = x^3 - 2x^2 + 7x$ at the origin.	
(h)	Show that $\sqrt{3}\sin x + 3\ \cos x$ has maximum for $= \frac{\pi}{6}$.	
3.	Answer any three questions:	5×3
(a)	Determine a, b, c such that $\lim_{x\to 0} \frac{x(a+b\ \cos x)+c\ x}{x^5} = \frac{1}{60}$	
(b)	A function $f(x)$ is defined as follows $f(x) = \frac{1}{2} - x$; when $0 < x < \frac{1}{2}$ $= \frac{1}{2}$; when $x = \frac{1}{2}$ $= \frac{3}{2} - x$; when $\frac{1}{2} < x < 1$ Show that, $f(x)$ is discontinuous at $= \frac{1}{2}$.	
(c)	Prove that, $\lim_{h\to 0} \frac{f(a+h)-2f(a)+f(a-h)}{h^2} = f''(a)$ Provided $f''(x)$ is continuous.	
(d)	Trace the curve $x^3 + y^3 = 3axy$.	

Q.No		Marks
(e)	Apply Maclaurin's theorem on $f(x) = (1+x)^4$ to deduce that, $$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4.$$	
4.	Answer any three questions:	10×1
(a)		
(i)	State and prove Cauchy's Mean Value theorem and deduce from it Lagrange's Mean Value theorem.	5+1
(ii)	Verify Cauchy's Mean Value theorem: $f(x) = e^x, g(x) = e^{-x}$ on [0,1]	2
(iii)	Is Rolle's theorem applicable on $f(x)$ in $[0, \pi]$ whrer $f(x) = \tan x$?	2
(b) (i)	If $y = \sin(m \sin^{-1} x)$, then show that $$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$	5
(ii)	If $u = \tan^{-1}(\frac{x^3+y^3}{x^2+y^2})$; prove that $$x\frac{\delta u}{\delta x} + y\frac{\delta u}{\delta y} = \frac{1}{2} Sin(2u).$$	5
(c)		
(i)	Show that the radius of curvature of the curve $x^4 + y^4 = 2$ at the point (1,1) is $\frac{\sqrt{2}}{3}$.	4
(ii)	Show that the pedal equation of the parabola $r = \frac{2a}{1-\cos\theta}$ is $p^2 = ar$.	4
(iii)	Verify that $f(x) = 2 + \lvert x \rvert$ has a minimum at $x = 0$.	2